# Sub-Partitions and Parking Functions

François Bergeron, UQAM

- COHOMOLOGY RING OF THE FLAG VARIETY
- MACDONALD POLYNOMIALS AND OPERATORS
- DIAGONAL HARMONICS
- DIAGONAL COINVARIANT SPACE
- HILBERT SCHEMES OF POINTS IN THE PLANE
- CHEREDNIK HECKE ALGEBRAS.
- REFINED KNOT INVARIANTS
- ELLIPTIC HALL ALGERA.
- RECTANGULAR CATALAN COMBINATORICS
- ....

# SUB-PARTITIONS

# (DYCK PATHS)

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# PARTITIONS

$\mu \vdash n$ $\quad |\mu| = n$

$$n = \mu_1 + \mu_2 + \cdots + \mu_k$$
$$\mu_i \geq \mu_{i+1} > 0$$

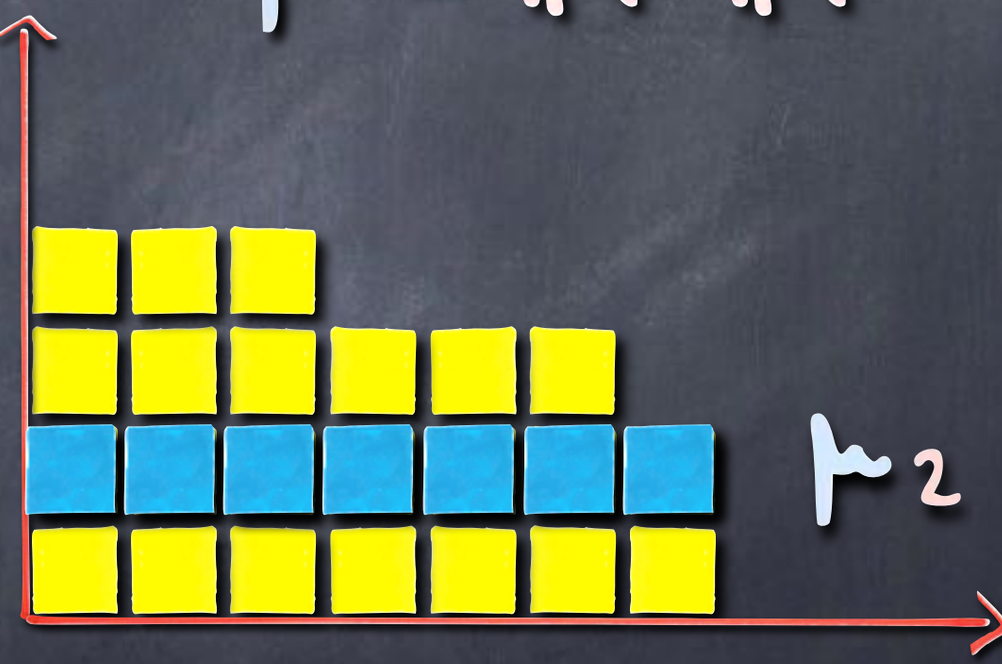
$\mu \subset \mathbb{N} \times \mathbb{N}$

$\mu = \mu_1 \mu_2 \cdots \mu_k$

$\mu = 1^{d_1} 2^{d_2} \cdots n^{d_n}$

$\mu$ HAS $d_i$ PARTS OF SIZE $i$

$\ell(\mu) = k = d_1 + d_2 + \cdots + d_n$

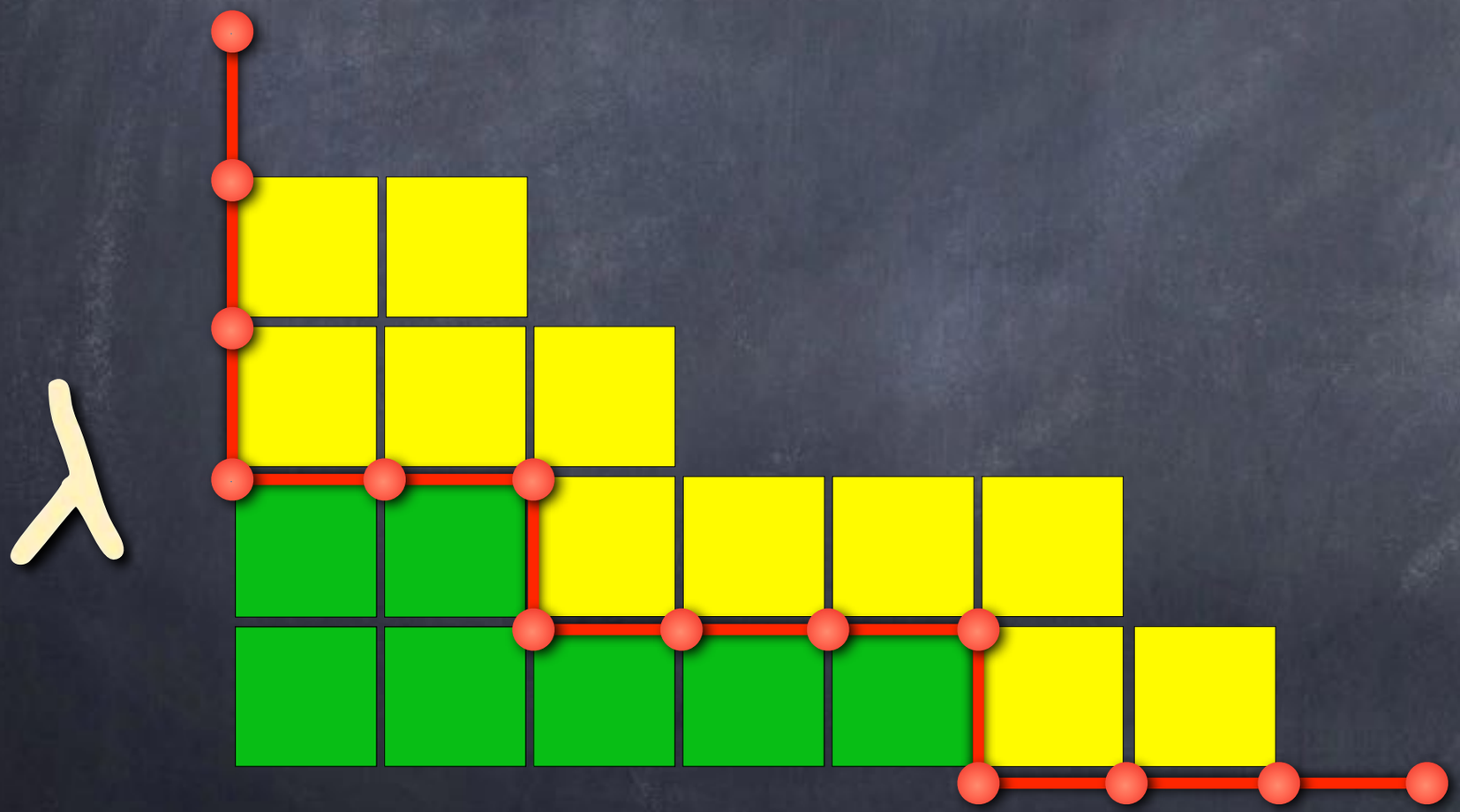



FERRERS DIAGRAM

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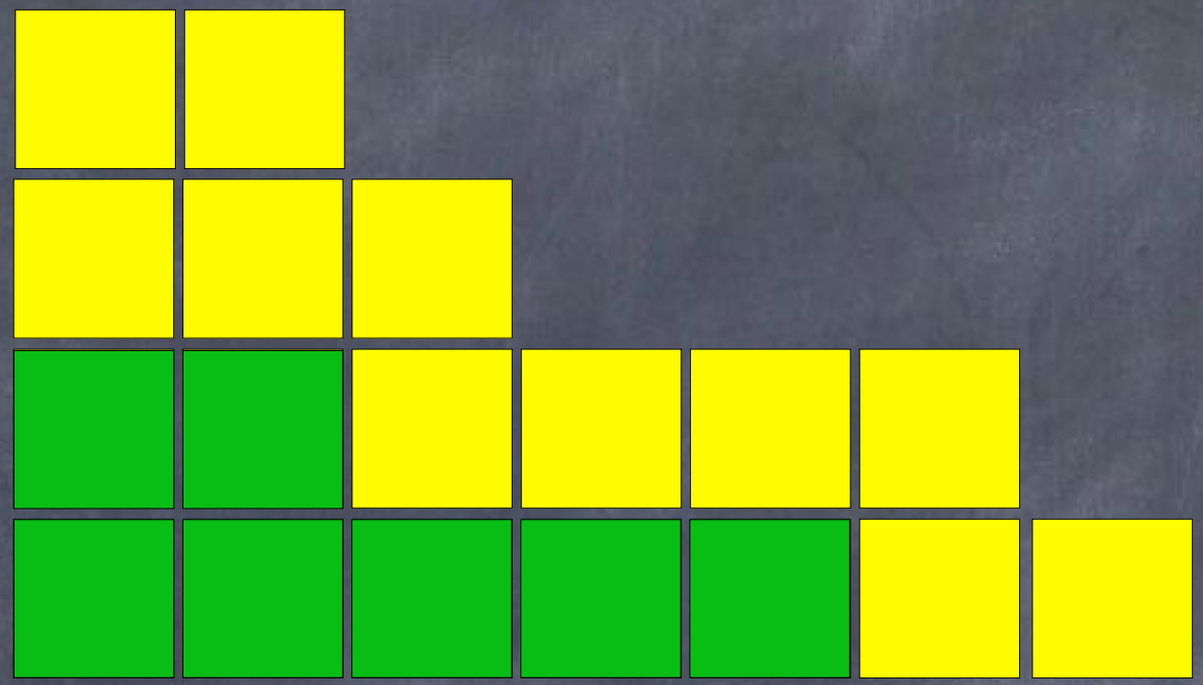
# SUB-PARTITIONS

$$\{\alpha \mid \alpha \subseteq \lambda\}$$




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# $\lambda$-Dyck Paths



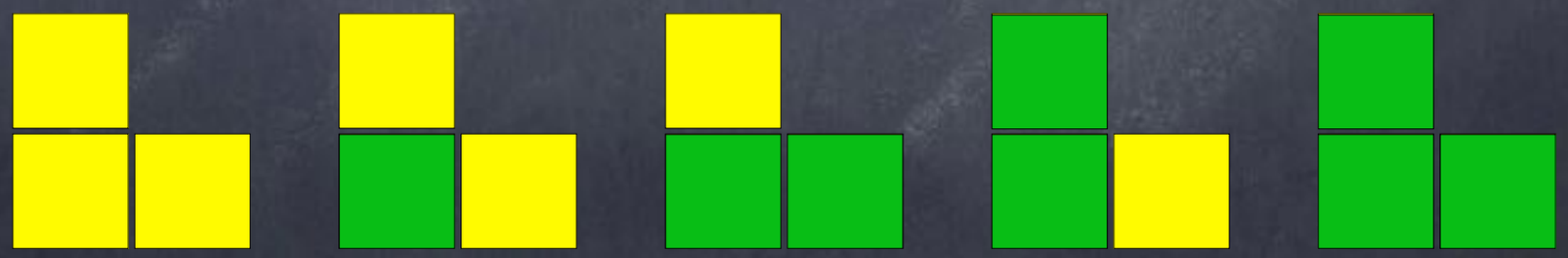
$\alpha \subseteq \lambda$

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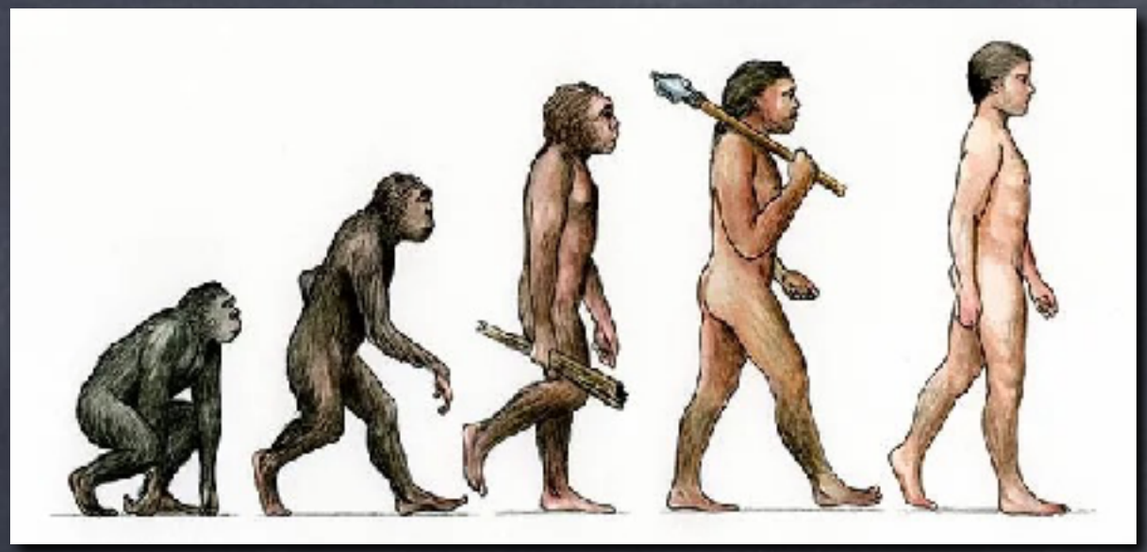
# $q$-Enumeration

$$A_\lambda(q) := \sum_{\alpha \subseteq \lambda} q^{|\lambda| - |\alpha|}$$

$$A_{\square\!\square}(q) := q^3 + q^2 + q + q + 1$$



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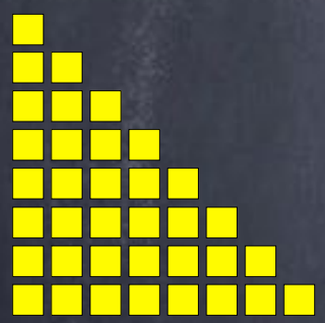


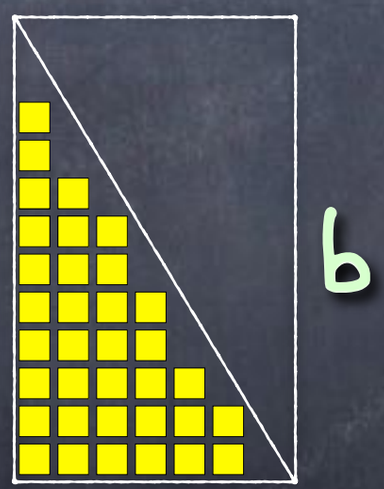
# Evolution of Catalan Combinatorics

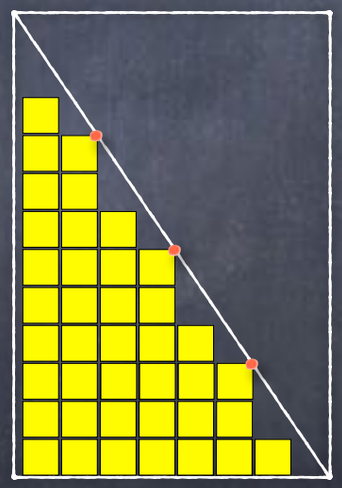


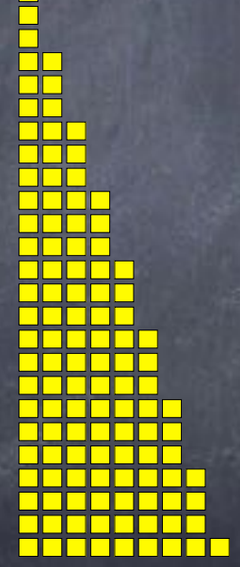
~1750 1954 2021

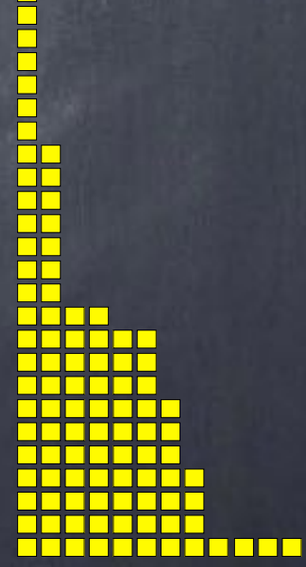
Classical ⊊ Rational ⊊ Rectangular ⊊ Triangular ⊊ Partitional









$a, b, k \in \mathbb{N} \quad (a,b) = 1$

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# Classical

# Rectangular

# Triangular

~1750

1793

1954

2021



LETTRE CXL.

EULER à GOLDBACH.

Berlin d. 4. September 1751.

So gross das Vergnügen auch ist, welches ich in Betrachtung der Eigenschaften der Zahlen finde, so wird mir doch diese Materie, wenn ich einige Zeit mit ganz andern Untersuchungen umgegangen, so fremd, dass ich mich sobald nicht mehr darin finden kann. Also konnte ich den Grund des schönen theorematis, dessen Ew. Meldung thun, dass eine summa quatuor quadratorum $aa+bb+cc+dd$ $=aa+bb+cc+(a+b+c+d)^2=$ $(a+b)^2+(a+c)^2+(b+c)^2$.



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SOLVTIO QVAESTIONIS,
QVOT MODIS POLYGONVM $n$ LATERVM
IN POLYGONA $m$ LATERVM,
PER DIAGONALES RESOLVI QVEAT.

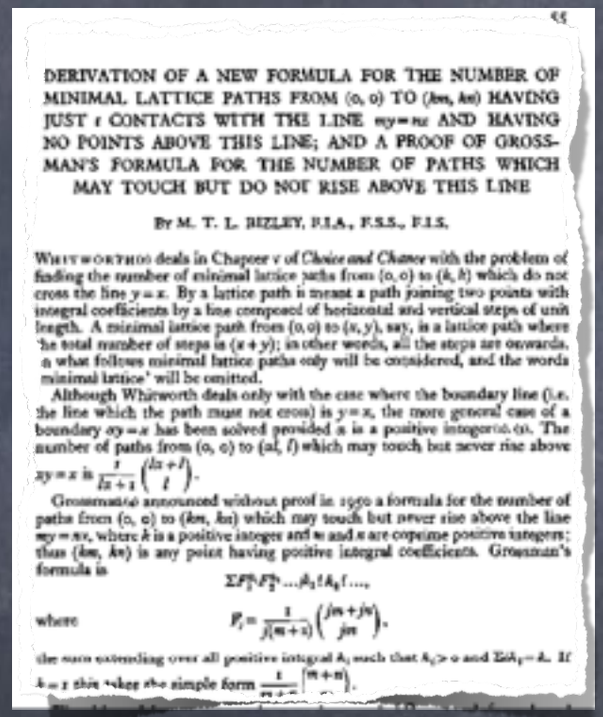
Auctore
NICOLAO FVSS.

Conventui exhib. die 9 Sept. 1793.

§. 1.

In litteris a Clarissimo Pfaff Helmstadio die $15^{to}$ mensis proxime superioris ad me datis, quibus acutissimus ille Geometra plura insignia inventa, integrationem potissimum formularum differentialium irrationalium spectantia, benevole mecum communicare voluit, mentio quoque fit Problematis: quot modis agonum in agona per diagonales resolvere liceat? cuius solutionem generalem se confecutum significabat Cl. Pfaff, quaerendo ex me, num mihi, praeter casum $m=3$, solutio aliqua huius quaestionis innotuerit. Cum igitur, praeter hunc casum memoratum, olim a Segnero in Tomo VII. novorum Commentationum tractatum, nulla mihi innotuerit solutio huius Problematis maxime curiosi, abstinere non potui, quominus ipse, quomodo inquisitio illa in genere suscipienda sit, tentarem. Methodum a me adhibitam, constructionemque meorum successus hic exhibere constitui.

§. 2.



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DERIVATION OF A NEW FORMULA FOR THE NUMBER OF MINIMAL LATTICE PATHS FROM (0, 0) TO (*kn*, *kn*) HAVING JUST *r* CONTACTS WITH THE LINE $ny=nx$ AND HAVING NO POINTS ABOVE THIS LINE; AND A PROOF OF GROSSMAN'S FORMULA FOR THE NUMBER OF PATHS WHICH MAY TOUCH BUT DO NOT RISE ABOVE THIS LINE

By M. T. L. BIZLEY, F.I.A., F.S.S., F.I.S.

WHITWORTH10 deals in Chapter v of *Choice and Chance* with the problem of finding the number of minimal lattice paths from (0, 0) to (*h*, *k*) which do not cross the line $y=x$. By a lattice path is meant a path joining two points with integral coefficients by a line composed of horizontal and vertical steps of unit length. A minimal lattice path from (0, 0) to (*x*, *y*), say, is a lattice path where the total number of steps is $(x+y)$; in other words, all the steps are onwards. In what follows minimal lattice paths only will be considered, and the words 'minimal lattice' will be omitted.

Although Whitworth deals only with the case where the boundary line (i.e. the line which the path must not cross) is $y=x$, the more general case of a boundary $xy=x$ has been solved provided $a$ is a positive integer(11). The number of paths from (0, 0) to (*al*, *l*) which may touch but never rise above $xy=x$ is $\frac{1}{la+1}\binom{la+l}{l}$.

Grossman(9) announced without proof in 1950 a formula for the number of paths from (0, 0) to (*kn*, *kn*) which may touch but never rise above the line $ny=nx$, where $k$ is a positive integer and $n$ and $s$ are coprime positive integers; thus (*kn*, *ks*) is any point having positive integral coefficients. Grossman's formula is

$$\Sigma P_{j_1}^{n_1} P_{j_2}^{n_2} \dots p_{j_1}! p_{j_2}! \dots,$$

where $P_j = \frac{1}{j(ns+s)}\binom{jns+js}{js}$,

the sum extending over all positive integral $j_i$ such that $j_i > 0$ and $\Sigma i j_i = k$. If $k=1$ this takes the simple form $\frac{1}{ns+s}\binom{ns+s}{s}$.



A SHUFFLE THEOREM FOR PATHS UNDER ANY LINE

J. HAGLUND, M. HAIMAN, J. HAGLUND, A. PUN, AND G. H. SEELINGER

Abstract. We generalize the shuffle theorem and its $(kn, ks)$ version, as conjectured by Haglund et al. and Bergeron et al., and proven by Carlsson and Mellit, and Mellit, respectively. In our version the $(kn, ks)$ Dyck paths on the combinatorial side are replaced by lattice paths lying under a line segment where $x$ and $y$ intercepts need not be integers, and the algebraic side is given either by a Schiffmann algebra operator formula or an equivalent explicit raising operator formula.

We derive our combinatorial identity as the polynomial truncation of an identity of infinite series of $GL_l$ characters, expressed in terms of infinite series versions of LLT polynomials. The series identity in question follows from a Cauchy identity for non-symmetric Hall-Littlewood polynomials.

1. Introduction

1.1. **Overview.** The shuffle theorem, conjectured by Haglund et al. [15] and proven by Carlsson and Mellit [6], is a combinatorial formula for the symmetric polynomial $\nabla e_k$ as a sum over LLT polynomials indexed by Dyck paths—that is, lattice paths from $(0, k)$ to $(k, 0)$ that lie weakly below the line segment connecting those two points. Here $e_k$ is the $k$-th elementary symmetric function, and $\nabla$ is an operator on symmetric functions with coefficients in $\mathbb{Q}(q, t)$ that arises in the theory of Macdonald polynomials [3].

The polynomial $\nabla e_k$ is significant because it describes the character of the ring $R_k$ of diagonal coinvariants for the symmetric group $S_k$ [18, Proposition 3.5]. The character of $R_k$ had been conjectured in the early 1990s to have surprising connections with the enumeration of combinatorial objects such as trees and Dyck paths—for instance, its dimension is equal to the number $(k+1)^{k-1}$ of rooted trees on $k+1$ labelled vertices, and the multiplicity of the sign character is equal to the Catalan number $C_k$. A summary of these early conjectures, attributed by a number of people, can be found in [19]. More precisely, $\nabla e_k$ describes the character of $R_k$ as a doubly graded $S_k$ module. The double grading in $R_k$ thus gives rise

LEONHARD PAUL EULER

NICOLAUS FUSS

BIZLEY

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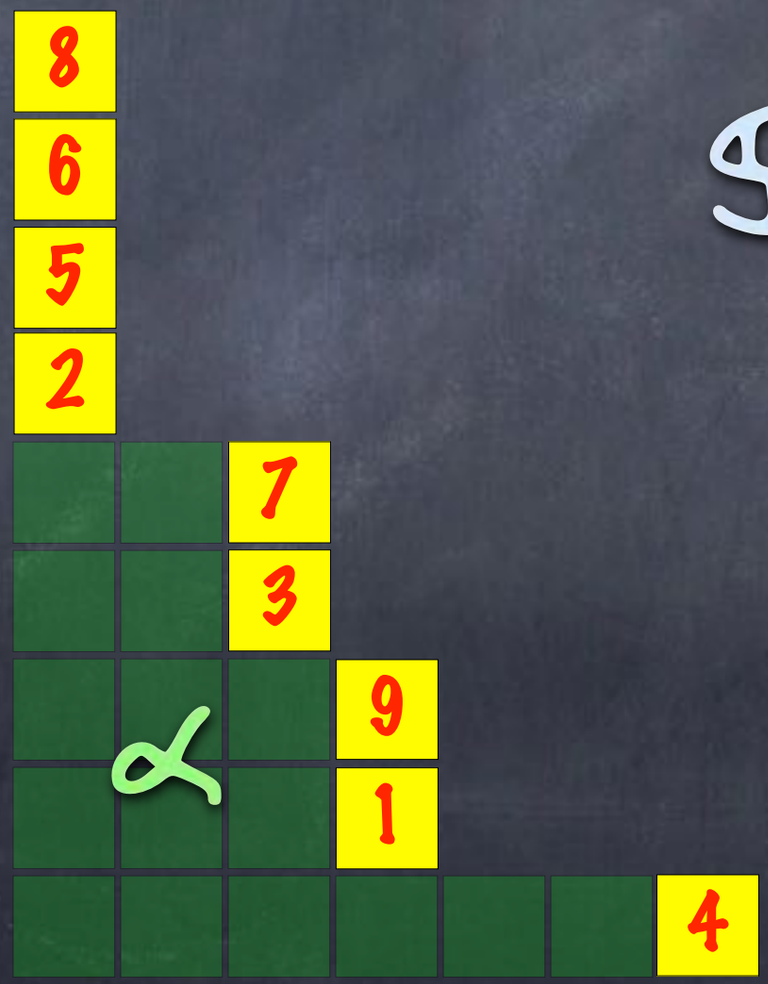
# Parking Functions

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# PARKING FUNCTION

STANDARD YOUNG TABLEAU

FORM $(\alpha + 1^m)/\alpha$




$\mathfrak{S}_m$ ACTION

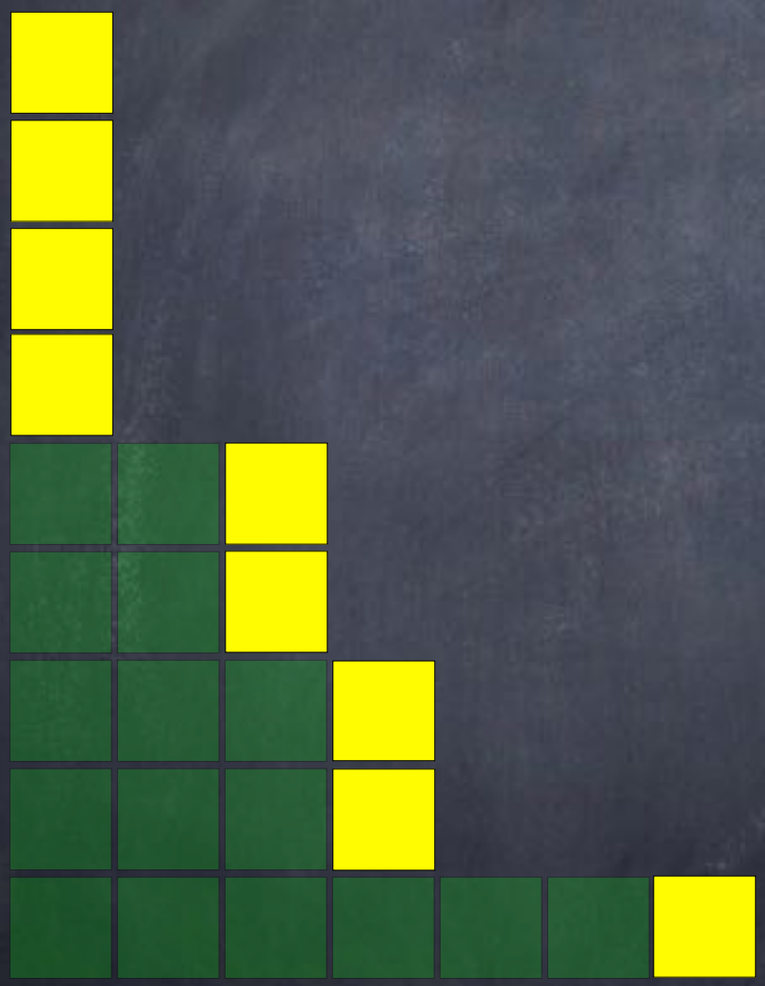
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# $\mathfrak{S}_n$ Action : Frobenius

$$\Delta_{(\alpha+1^m)/\alpha}$$



# $\mathfrak{S}_n$ Action : Frobenius



$$\mathfrak{S}_{(\alpha+1^m)/\alpha} = e_4 e_2^2 e_1$$

# TRIANGULAR PARTITIONS (UNDER ANY LINE)

IN COLLABORATION WITH
MIKHAIL MAZIN

ARXIV: 2203.15942

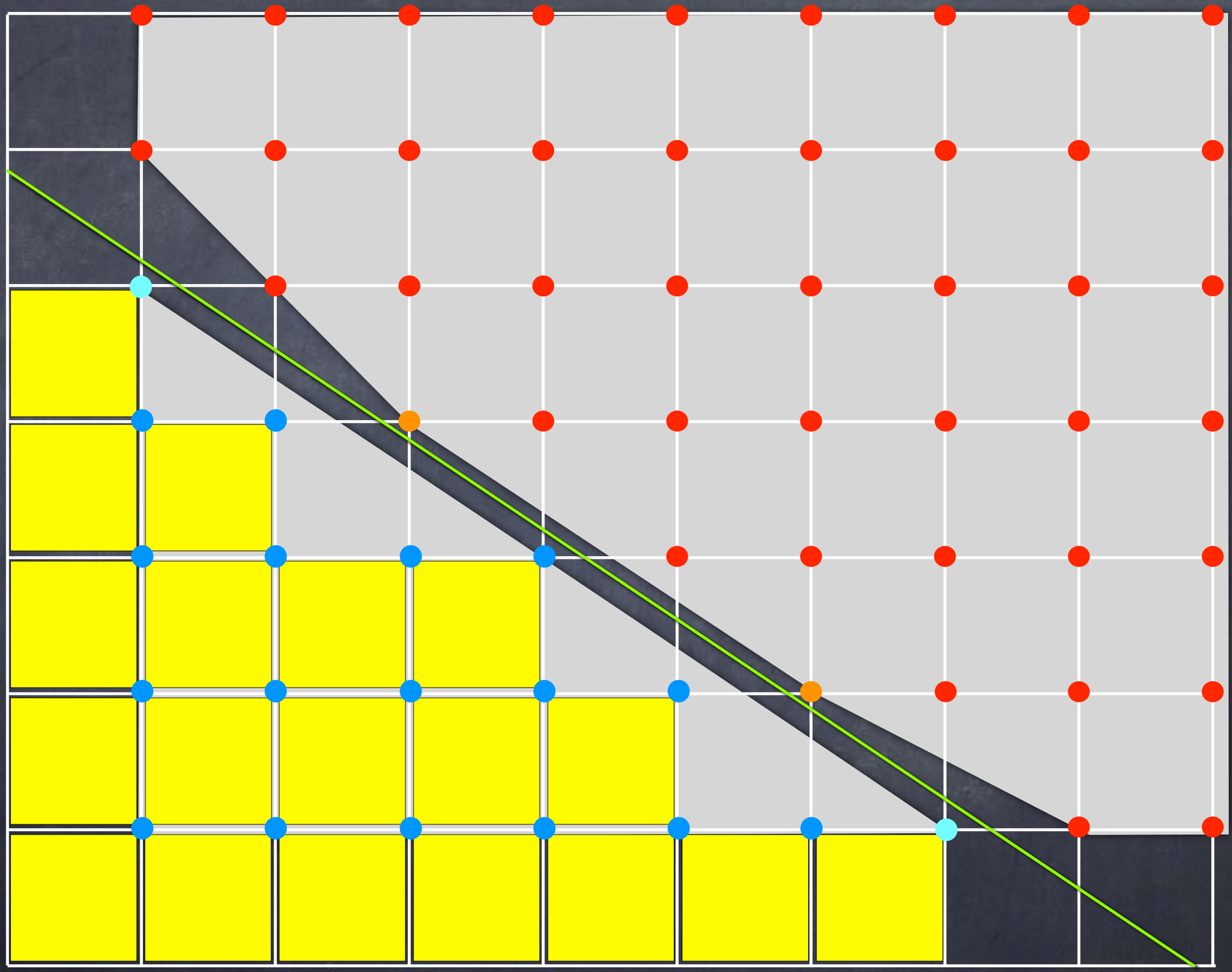
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# BLASIAK, HAIMAN, MORSE PUN, SEELINGER

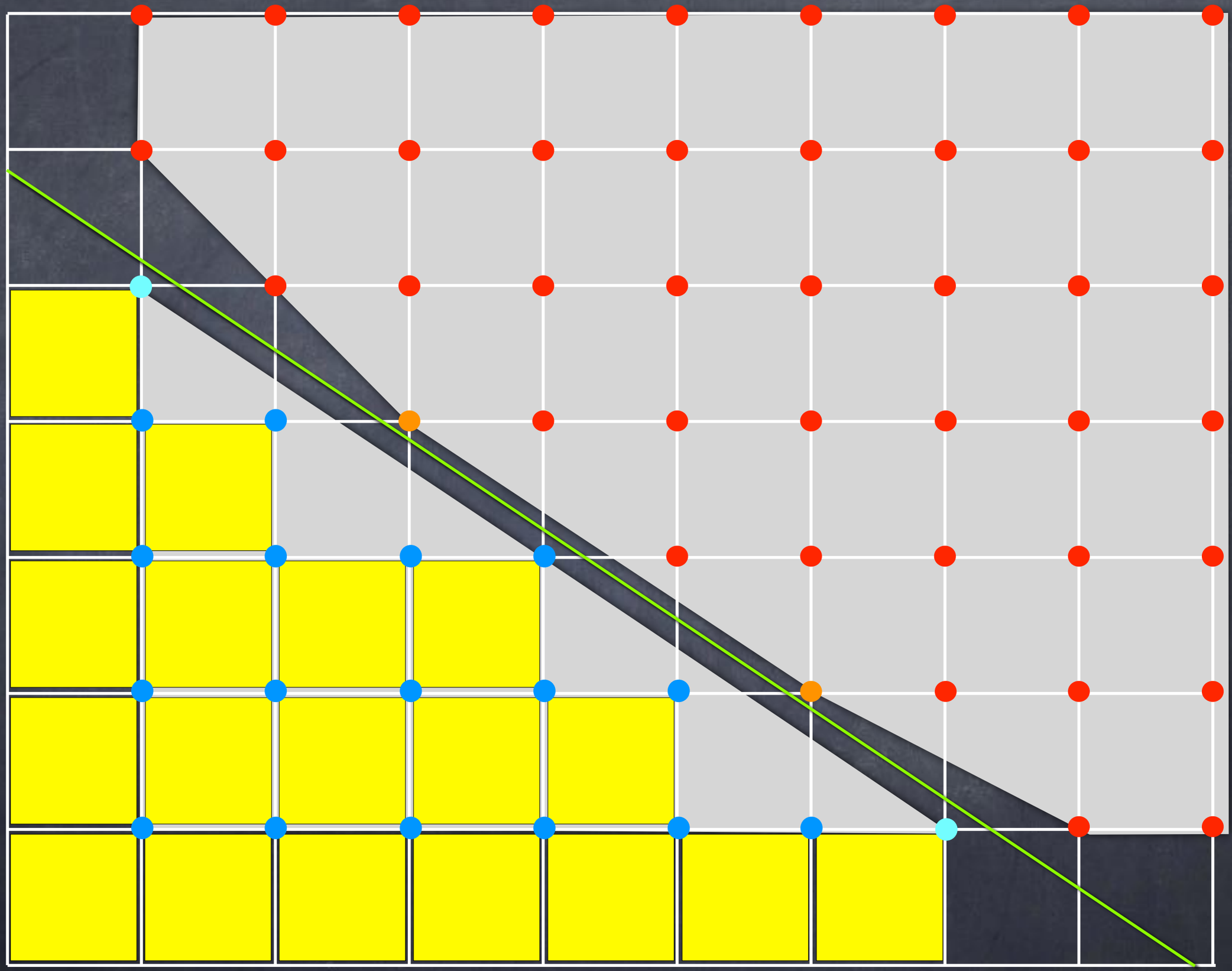
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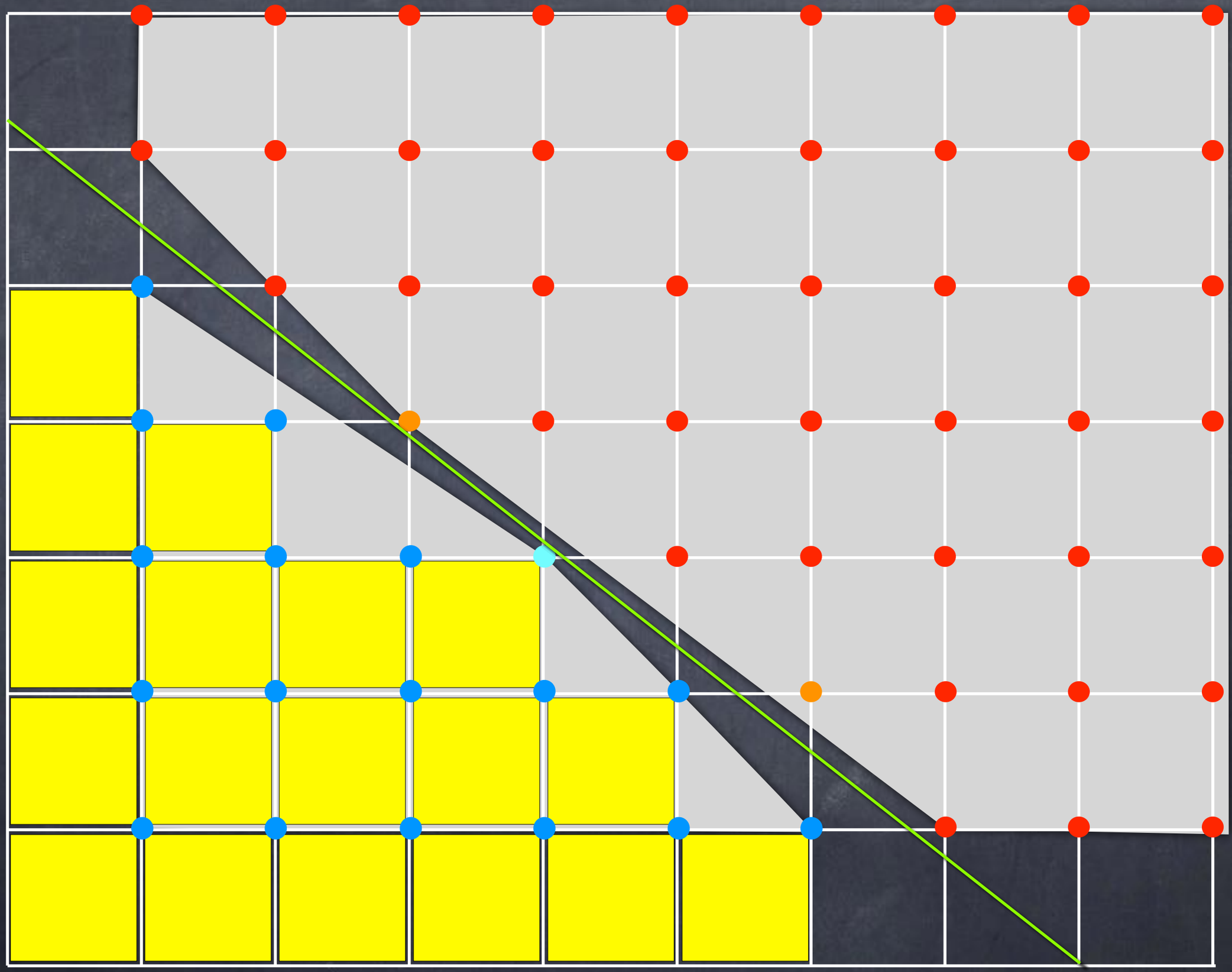
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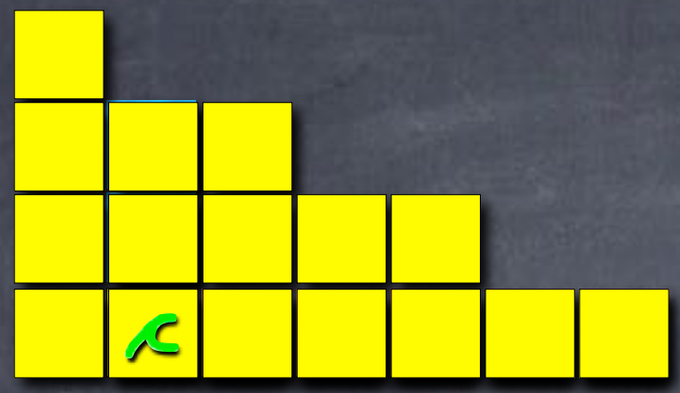
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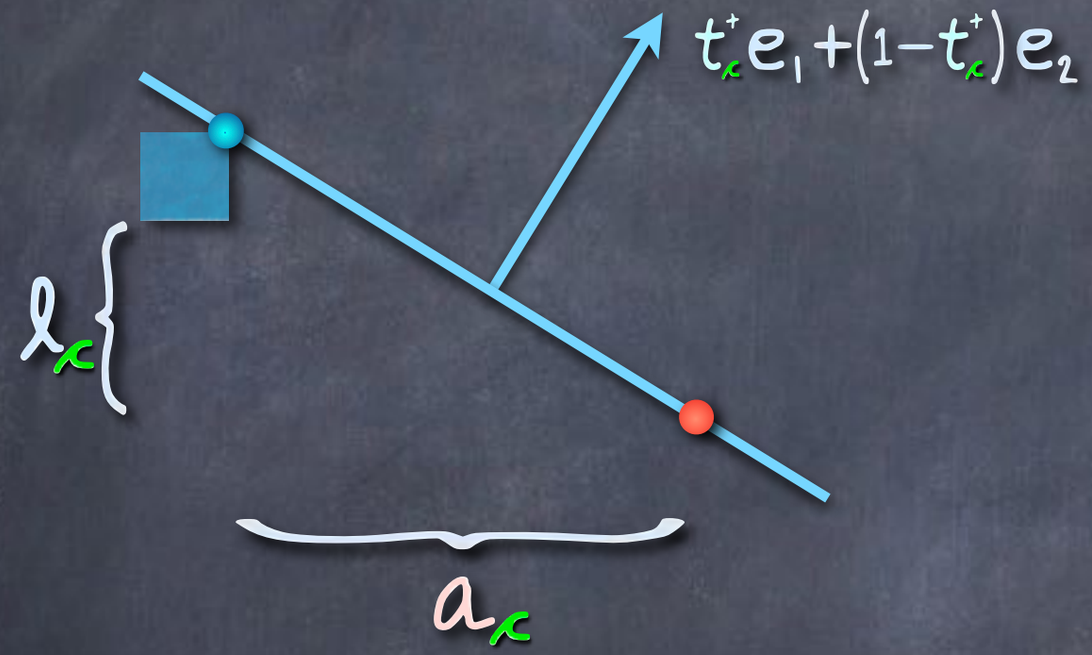
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# Triangular Partition

$\tau$




$\tau \vdash m$

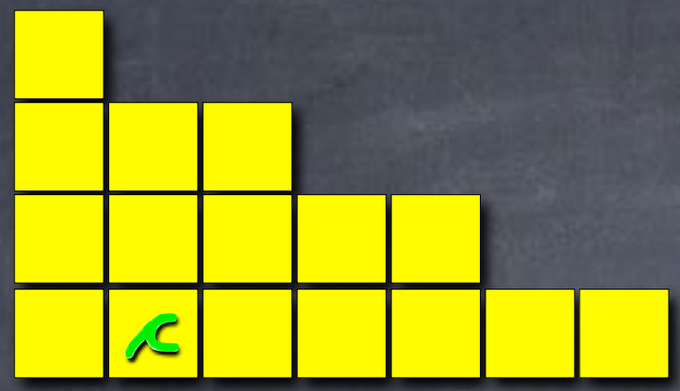



$$t_\kappa^+ := \frac{\ell_\kappa + 1}{a_\kappa + \ell_\kappa + 1}$$

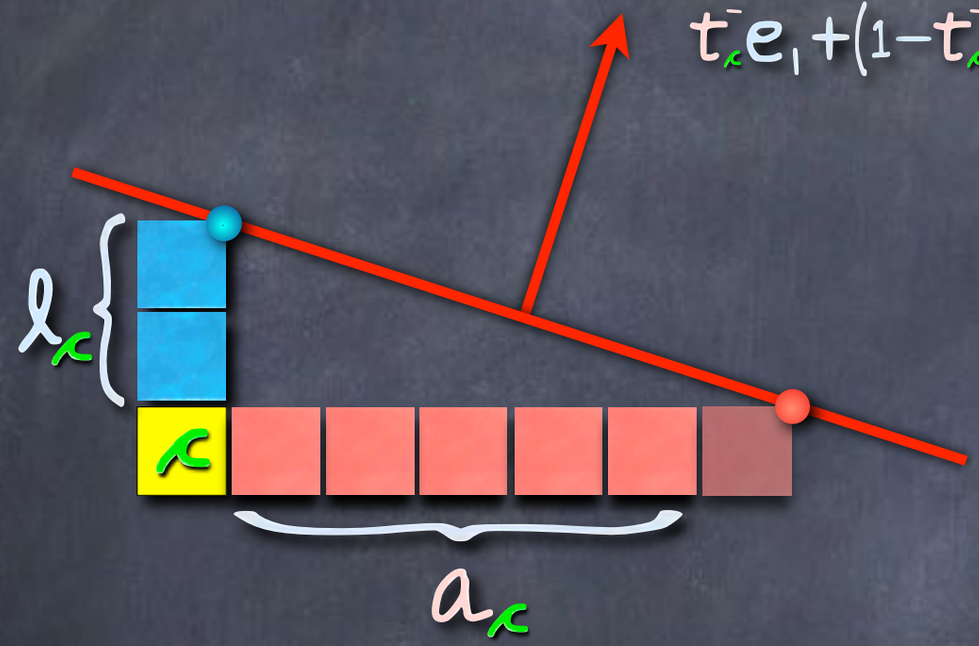
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# TRIANGULAR PARTITION

$\tau$




$\tau \vdash m$




$$t_\kappa^+ := \frac{\ell_\kappa + 1}{a_\kappa + \ell_\kappa + 1}$$

$$t_\kappa^- := \frac{\ell_\kappa}{a_\kappa + \ell_\kappa + 1}$$

**LEMMA**

$\tau$ TRIANGULAR IFF $\max_{\kappa \in \tau} t_\kappa^- < \min_{\kappa \in \tau} t_\kappa^+$

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# Triangular partition

$t^-_\tau := \max_{\kappa \in \tau} t^-_\kappa$

$\min_{\kappa \in \tau} t^+_\kappa =: t^+_\tau$

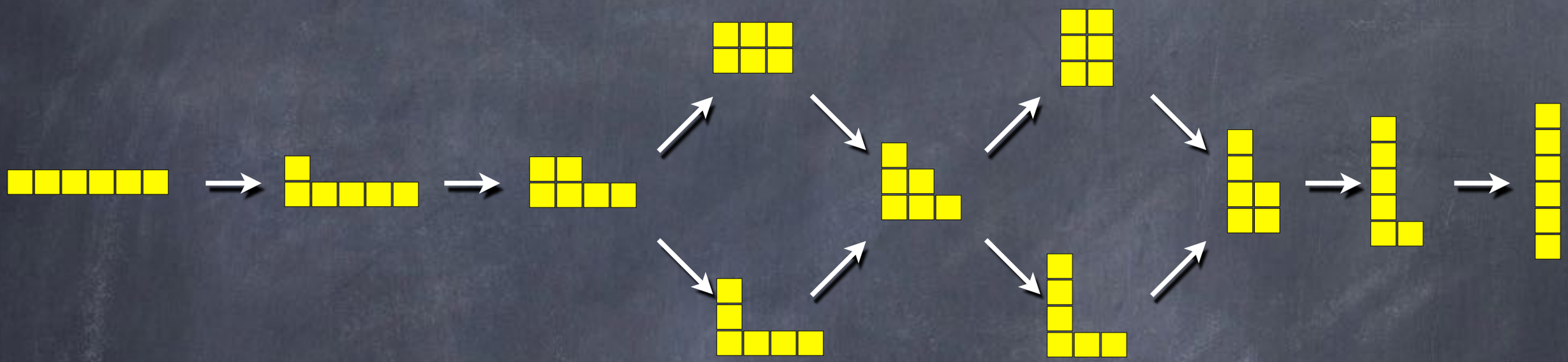
$\tau$

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# Orders

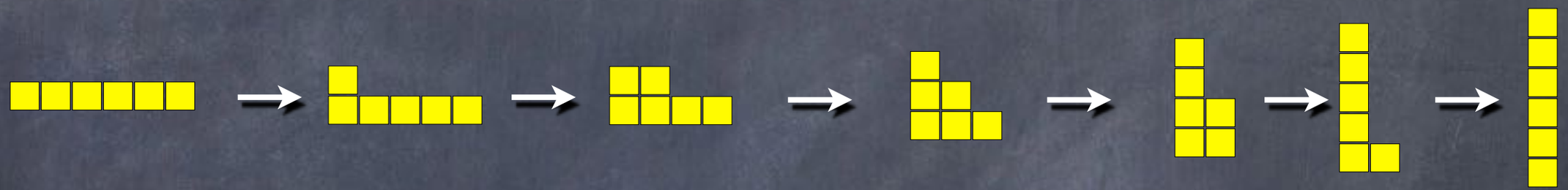
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# Dominance Order



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# Triangular Dominance Order

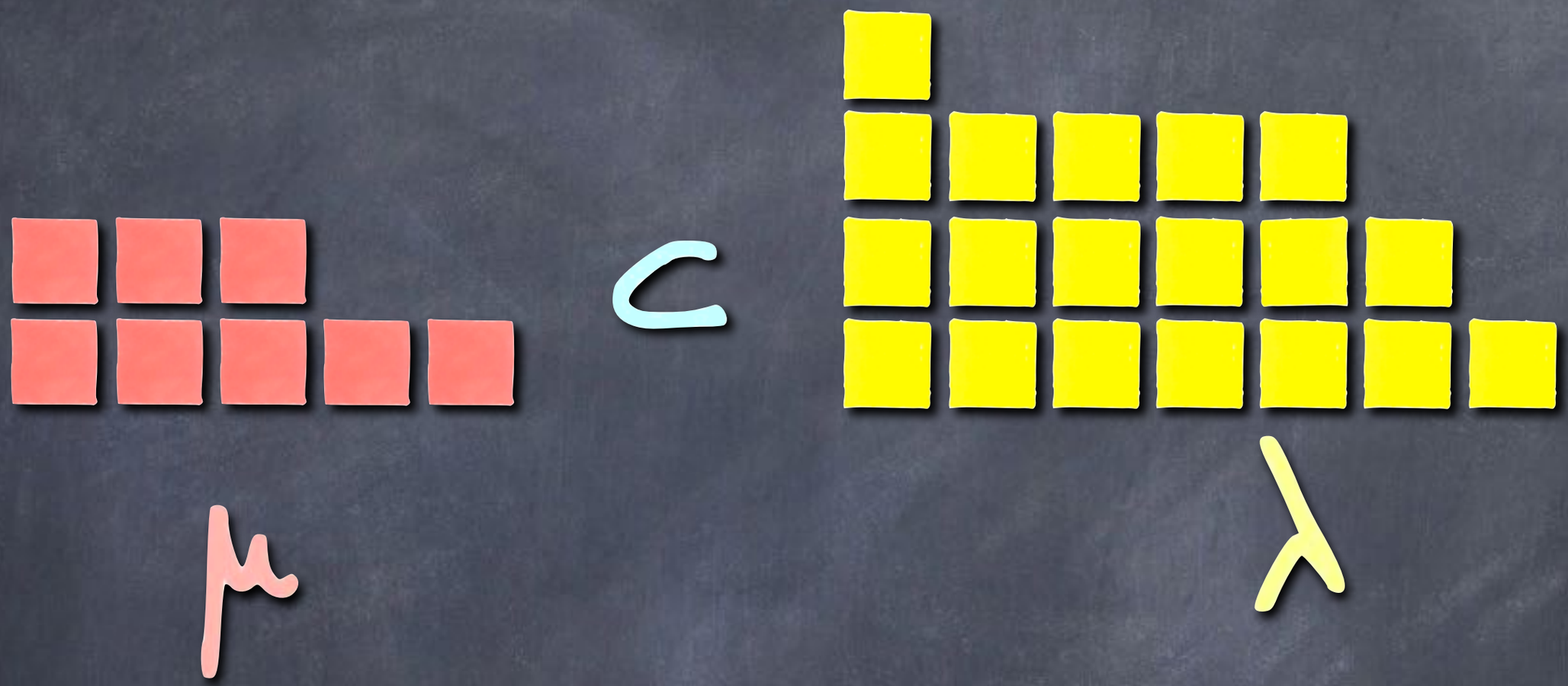


$(t_\tau^-, t_\tau^+)$ $(0, \frac{1}{6})$ $(\frac{1}{6}, \frac{1}{4})$ $(\frac{1}{4}, \frac{2}{5})$ $(\frac{2}{5}, \frac{3}{5})$ $(\frac{3}{5}, \frac{3}{4})$ $(\frac{3}{4}, \frac{5}{6})$ $(\frac{5}{6}, 1)$

# Total Order

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# CONTAINMENT ORDER



$\mu$ $\subset$ $\lambda$

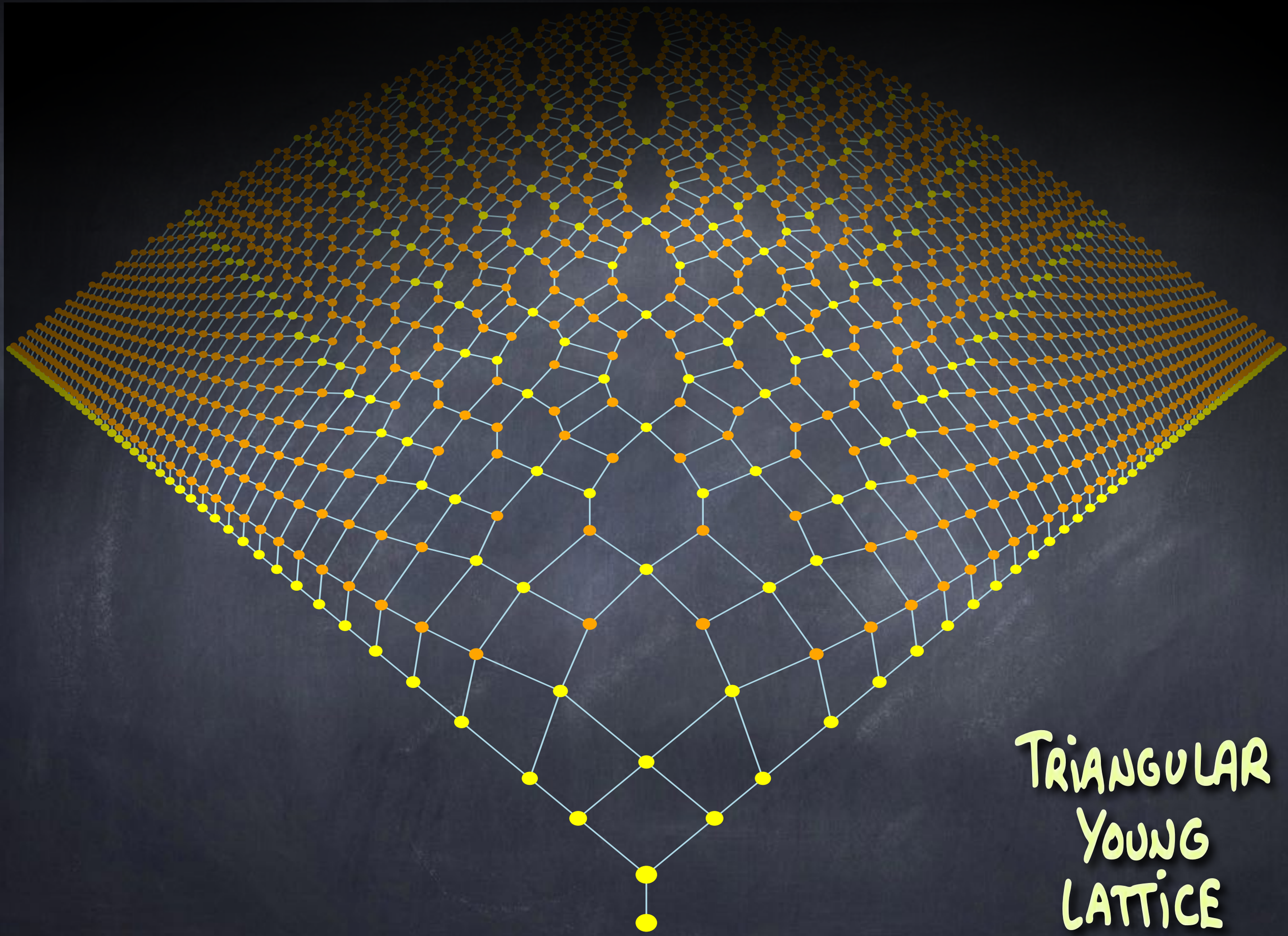
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# Triangular Young Lattice





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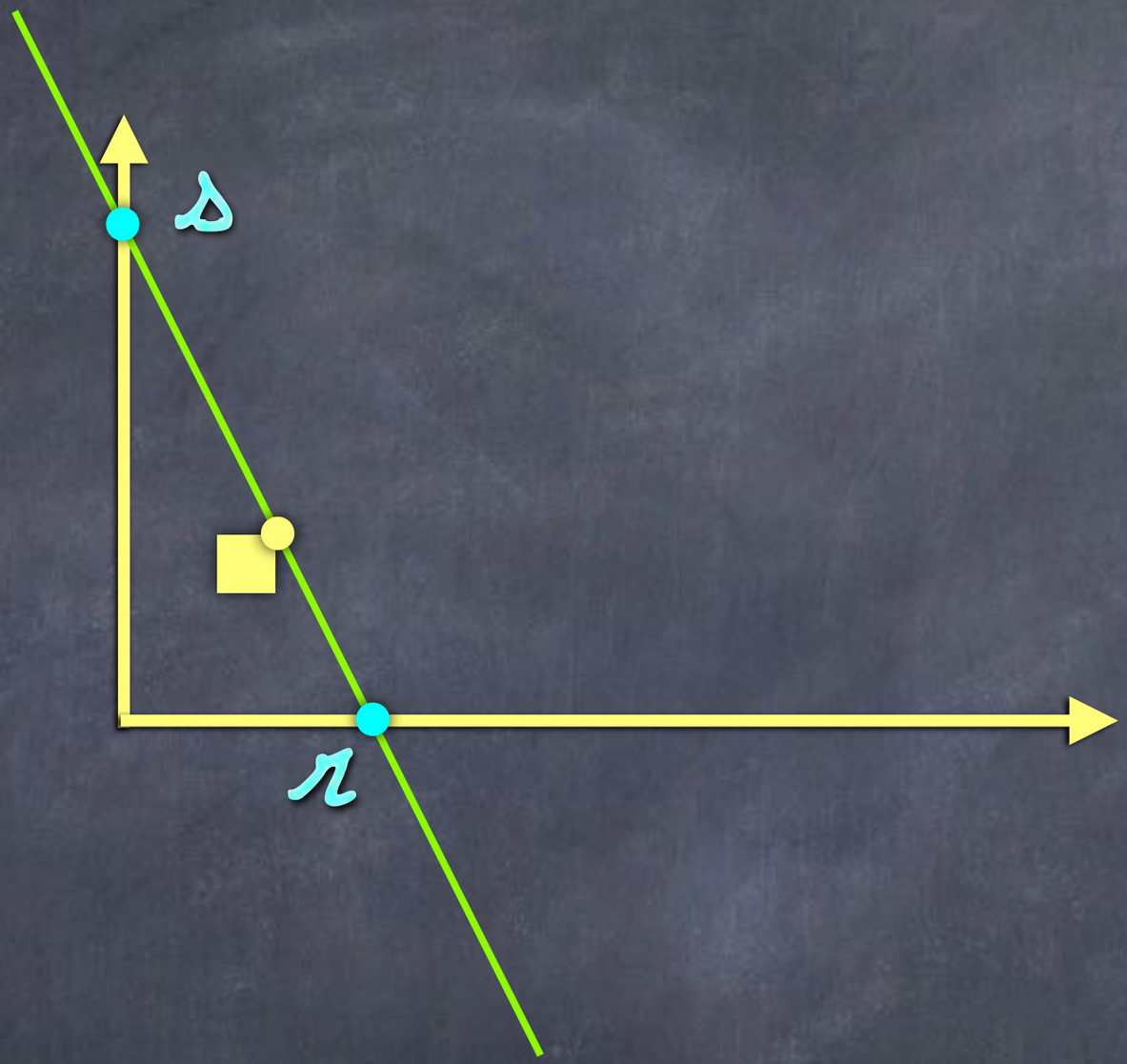
# TRIANGULAR YOUNG LATTICE

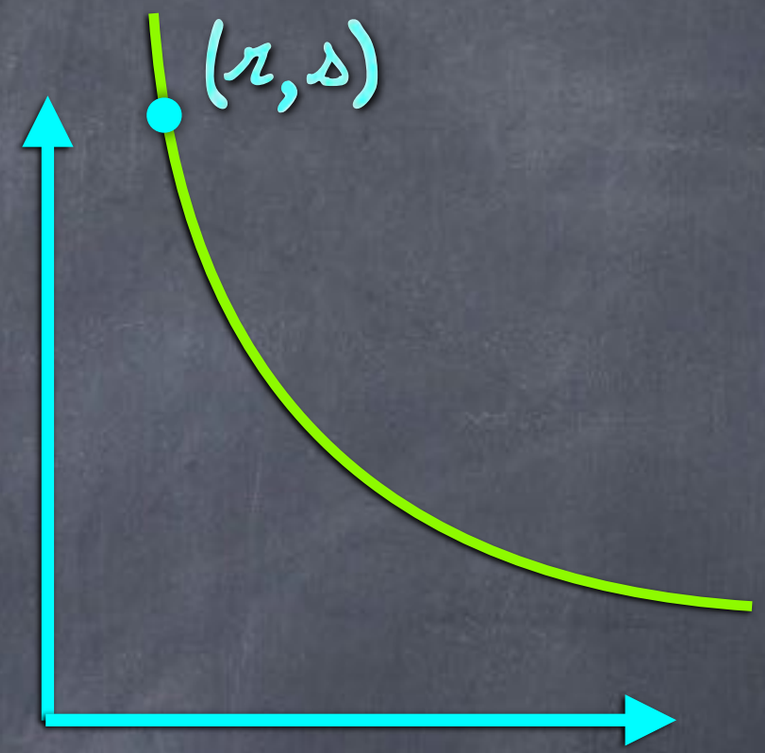
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# THEOREM

## HASSE DIAGRAM OF  IS PLANAR

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s
π

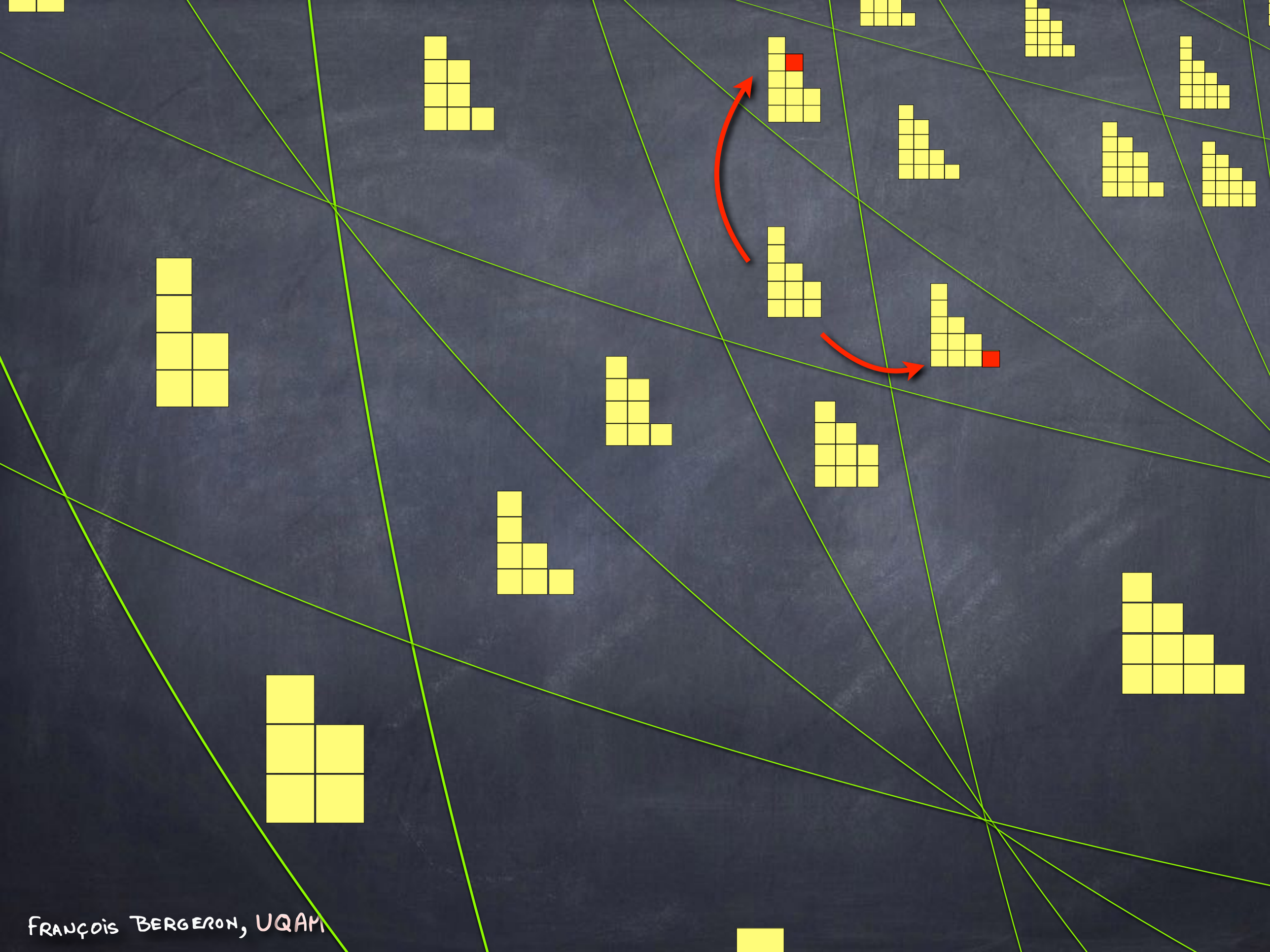

(π, s)

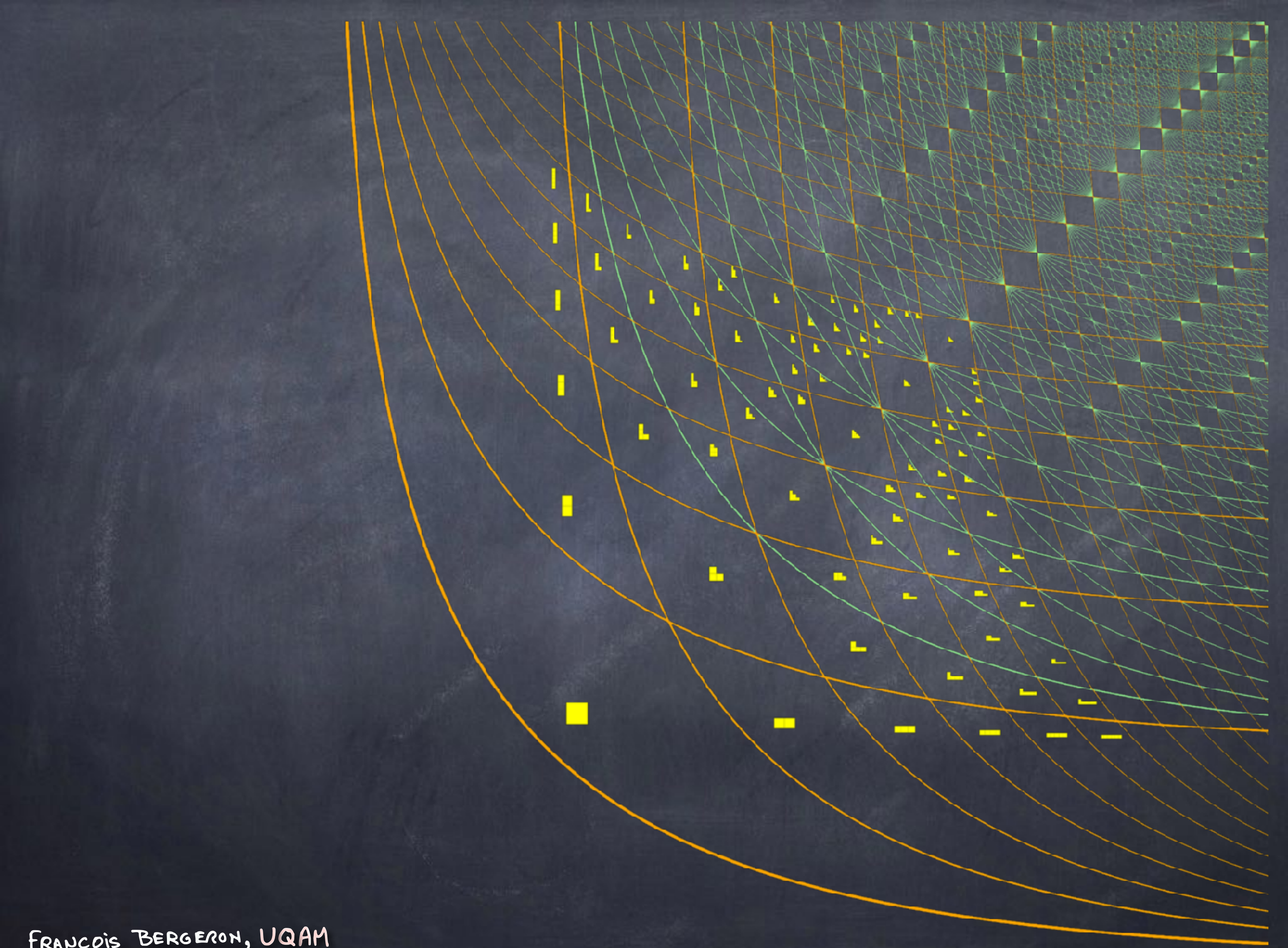
$\delta$

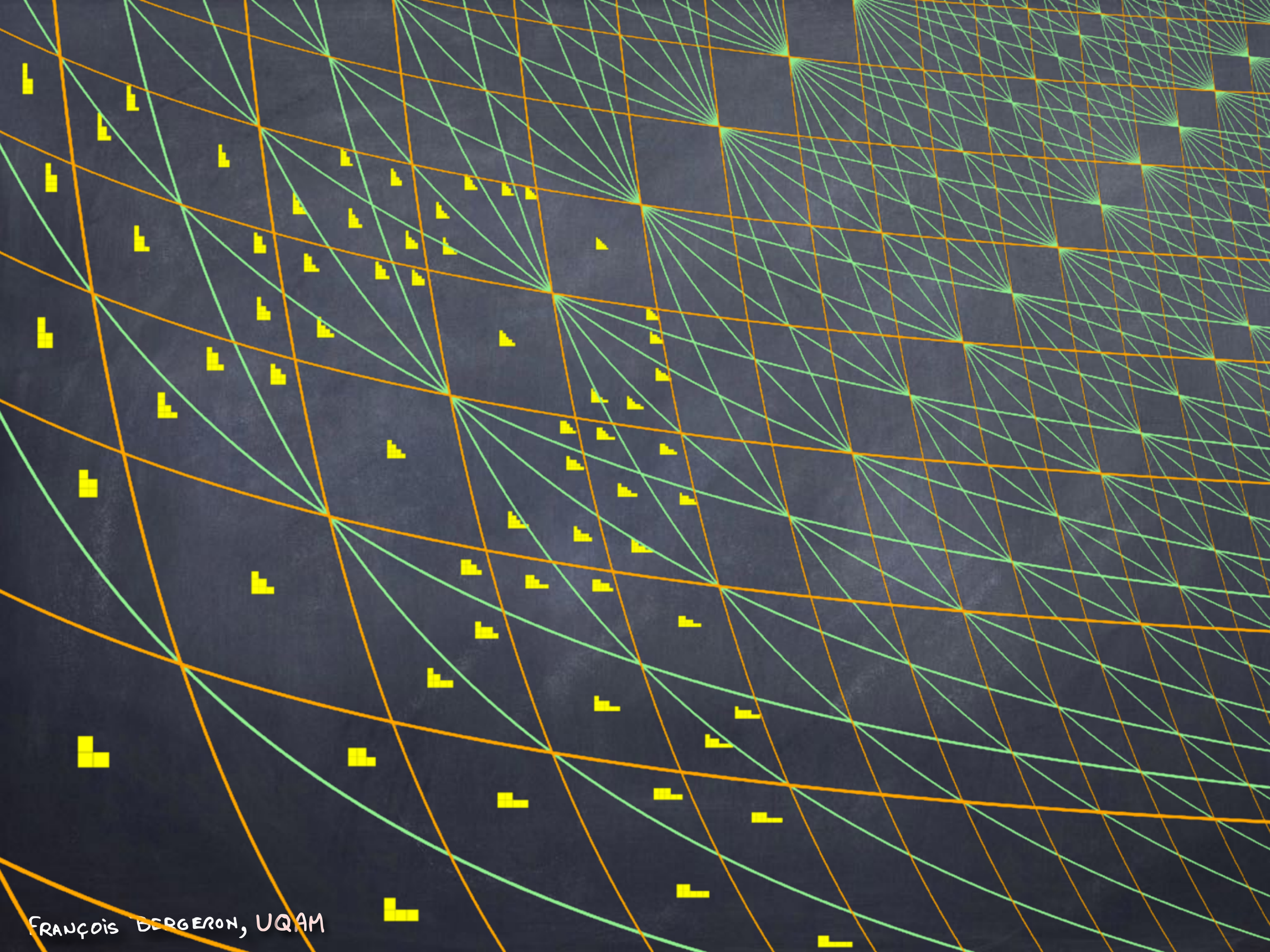
$\phi$

$n$

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# Triangular Dyck Paths

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# $q$-ENUMERATION

**PROPOSITION**

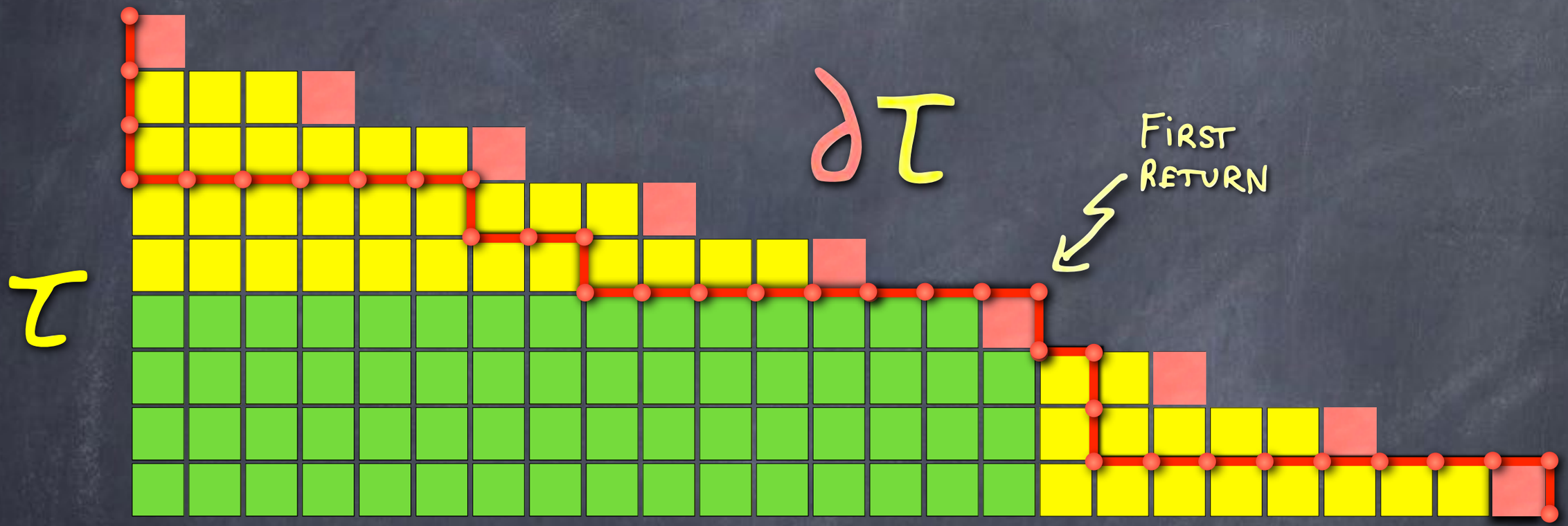
$$A_\tau(q) = \sum_{(\alpha,\beta)} q^{|\alpha \setminus \partial\tau|} A_\alpha(q) A_\beta(q)$$

$$A_\varepsilon(q) = 1$$

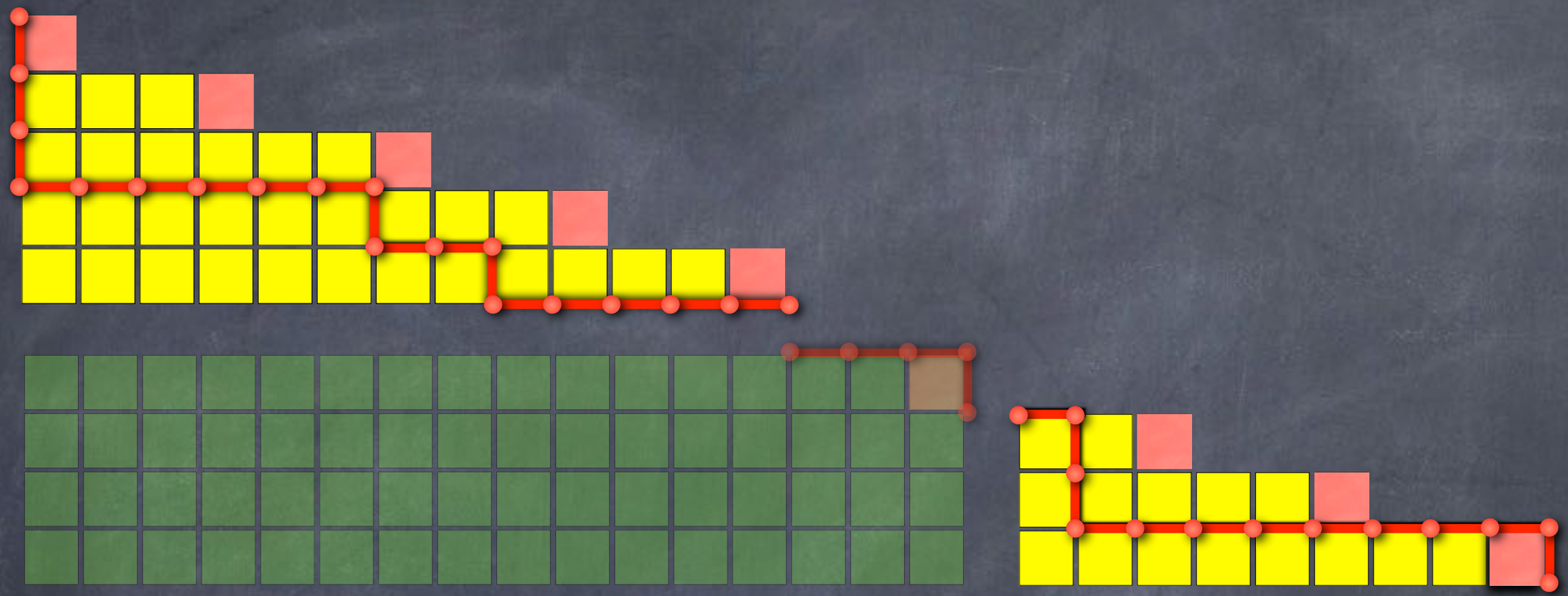
## PROOF: FIRST RETURN TO DIAGONAL $\partial\tau$

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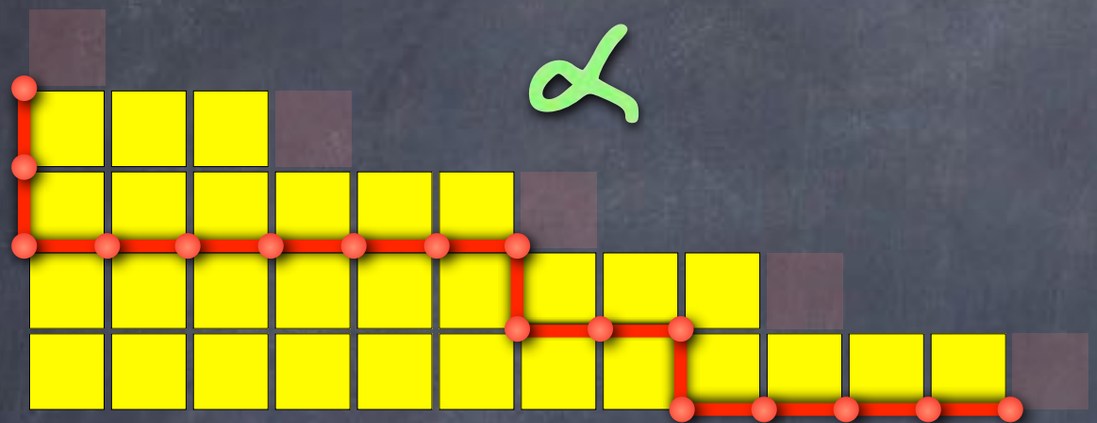
# DIAGONAL OF $\tau$

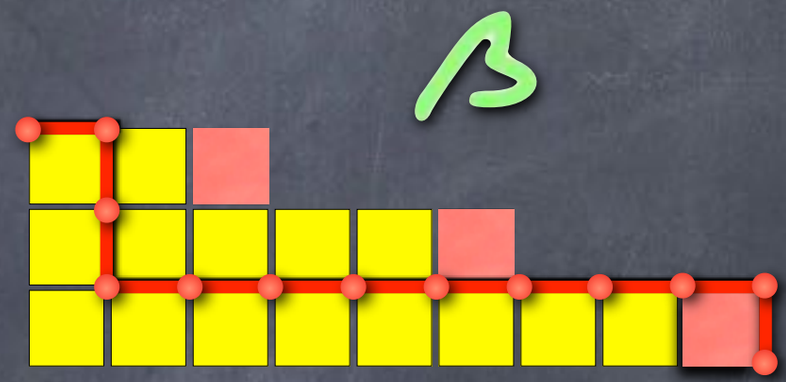



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# $(q,t)$-Enumeration

$$A_\tau(q,t) := \sum_{\alpha \subseteq \tau} q^{|\tau|-|\alpha|}\, t^{\mathrm{dinv}_\tau(\alpha)}$$

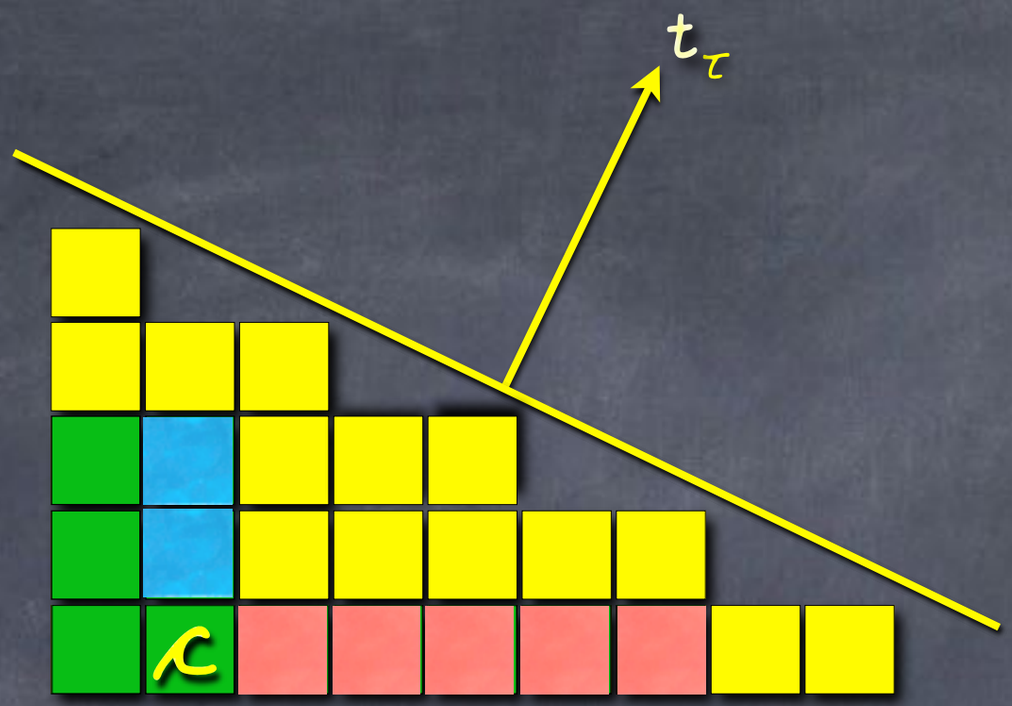
$$A_{\square\!\square\!\square}(q,t) = q^3 + q^2 t + q t + q t^2 + t^3$$

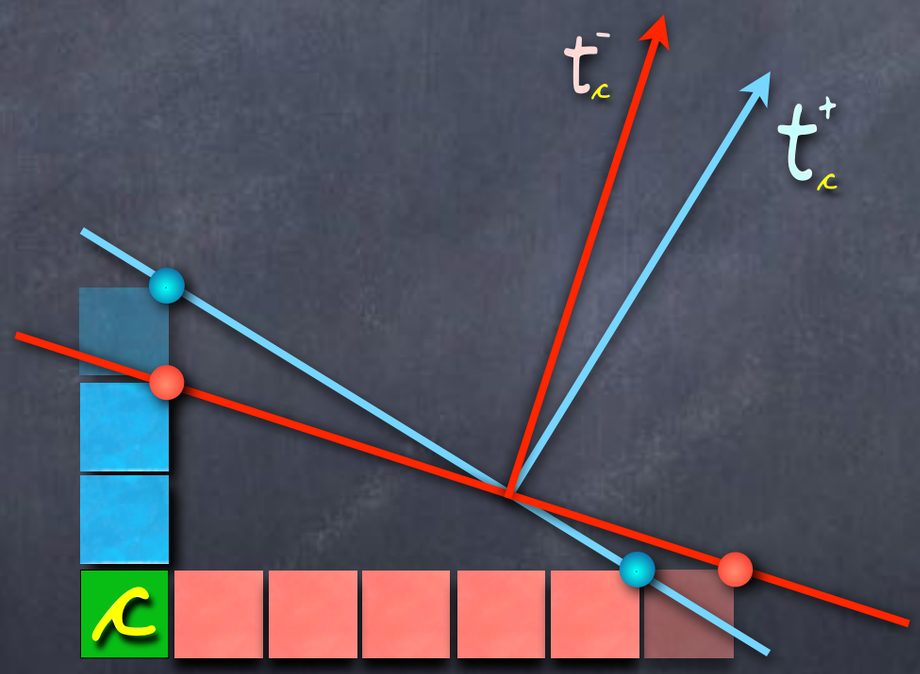


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# DINV CELLS

$\alpha \subseteq \tau$



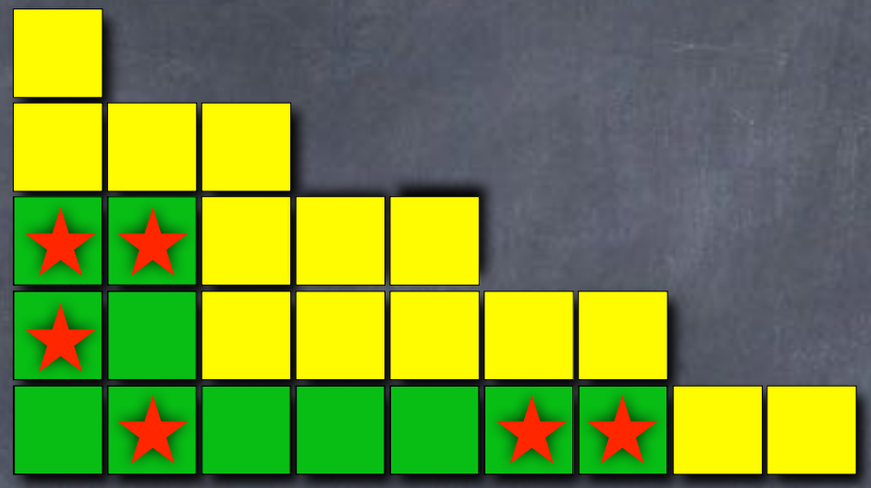

$$t_c^+ := \frac{l_c + 1}{a_c + l_c + 1}$$

$$t_c^- := \frac{l_c}{a_c + l_c + 1}$$

$t_c^-$, $t_c^+$ { FOR HOOK OF $c$ IN $\alpha$

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$\alpha \subseteq \tau$



$$\mathrm{DINV}_\tau(\alpha) := \left\{ \kappa \;\middle|\; t^-_\kappa \le t_\tau < t^+_\kappa \right\}$$

$$\mathrm{dinv}_\tau(\alpha) := \#\,\mathrm{DINV}_\tau(\alpha)$$

$$t_\tau := \frac{t^-_\tau + t^+_\tau}{2}$$

# NEGUT FORMULA

$$A_I(q,t) = \sum_{\mu \vdash m} \left( \sum_{\theta \in SYT(\mu)} z_\tau(\theta)\, \Omega(\theta) \right) \prod_{\kappa \in \mu} \mathcal{Z}_\kappa$$

$$\mathcal{Z}_\kappa := q^i t^j \qquad \kappa = (i,j)$$

$$\Omega(\theta) = \prod_{\theta(\kappa)<\theta(b)} \frac{(\mathcal{Z}_b - \mathcal{Z}_\kappa)^*(\mathcal{Z}_b - qt\mathcal{Z}_\kappa)^*}{(\mathcal{Z}_b - q\mathcal{Z}_\kappa)^*(\mathcal{Z}_b - t\mathcal{Z}_\kappa)^*} \prod_{\theta(b)=\theta(\kappa)+1} \frac{\mathcal{Z}_b}{(\mathcal{Z}_b - qt\mathcal{Z}_\kappa)} \prod_{\kappa \in \mu} \frac{1}{\mathcal{Z}_\kappa - 1}$$

* AVOIDING 0 FACTORS

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# NEGUT FORMULA

$$\Omega(\theta) = \prod_{\theta(c)<\theta(b)} \frac{(z_b - z_c)^*(z_b - qtz_c)^*}{(z_b - qz_c)^*(z_b - tz_c)^*}$$

$$\prod_{\theta(b)=\theta(c)+1} \frac{z_b}{(z_b - qtz_c)} \prod_{c\in\mu} \frac{1}{z_c - 1}$$

$$Z_\tau(\theta) = \prod_{c\in\mu} z_c^{\varepsilon(\mu,\theta(c))}$$

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$$A_{\ydiagram{2,1}}(q,t) = \underbrace{q^3 + q^2 t + q t^2}_{} + t^3 + q t$$

$$= s_3(q,t) + s_{111}(q,t)$$

**CONJECTURE**

FOR ALL TRIANGULAR PARTITIONS $\tau$,

$A_\tau(q,t)$ IS SCHUR POSITIVE.

(SOME CASES SHOWN)

$$A_{\ydiagram{2,1}}(q,t) = \underbrace{q^3 + q^2 t + q t^2 + t^3} + q t$$

$$= s_3(q,t) + s_{111}(q,t)$$

**CONJECTURE NEGUT**

FOR ALL CONCAVE PARTITIONS $\tau$,

$A_\tau(q,t)$ IS SCHUR POSITIVE.

(SOME CASES SHOWN)

# GL-ACTION

$$A_T = A_T(q_1, q_2, q_3, \ldots)$$

# HILBERT SERIES OF ALTENATING COMPONENT OF MULTIVARIATE DIAGONAL HARMONIC POLYNOMIALS

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# GL-ACTION

$$A_T(q_1, q_2, q_3, \dots) = \sum_\lambda a_\lambda \Lambda_\lambda(q_1, q_2, q_3, \dots)$$

$$a_\lambda \in \mathbb{N}$$

$$A_T(q, t) = \sum_\lambda a_\lambda \Lambda_\lambda(q, t)$$

↑

AT MOST TWO PARTS

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$$A_\tau = \sum_\lambda a_\lambda \Lambda_\lambda$$

$$A_0 = \Lambda_0, \qquad A_1 = \Lambda_1, \qquad A_2 = \Lambda_2, \qquad A_3 = \Lambda_3,$$

$$A_{21} = \Lambda_3 + \Lambda_{11}, \quad A_{31} = \Lambda_4 + \Lambda_{21}, \quad A_{41} = \Lambda_5 + \Lambda_{31}, \quad A_{51} = \Lambda_6 + \Lambda_{41},$$

$$A_{32} = \Lambda_5 + \Lambda_{31}, \qquad A_{42} = \Lambda_6 + \Lambda_{41} + \Lambda_{22},$$

$$A_{52} = \Lambda_7 + \Lambda_{51} + \Lambda_{32}, \qquad A_{62} = \Lambda_8 + \Lambda_{61} + \Lambda_{42},$$

$$A_{93} = \Lambda_{12} + \Lambda_{82} + \Lambda_{63} + \Lambda_{10,1},$$

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$$A_\tau = \sum_\lambda a_\lambda \Lambda_\lambda$$

$$A_{321} = \Lambda_6 + \Lambda_{41} + \Lambda_{31} + \Lambda_{111},$$

$$A_{421} = \Lambda_7 + \Lambda_{51} + \Lambda_{41} + \Lambda_{32} + \Lambda_{211},$$

$$A_{431} = \Lambda_8 + \Lambda_{61} + \Lambda_{51} + \Lambda_{42} + \Lambda_{311},$$

$$A_{4321} = \Lambda_{10} + \Lambda_{81} + \Lambda_{71} + \Lambda_{61} + \Lambda_{62} + \Lambda_{42} + \Lambda_{43} + \Lambda_{311} + \Lambda_{411} + \Lambda_{511} + \Lambda_{1111},$$

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$$A_{54321} = s_{11111} + s_{3111} + s_{4111} + s_{5111} + s_{6111}$$
$$+ s_{611} + s_{711} + 2s_{811} + s_{911} + s_{10.11}$$
$$+ s_{421} + s_{521} + s_{621} + s_{721} + s_{821}$$
$$+ s_{431} + s_{531} + s_{631} + s_{441}$$
$$+ s_{10.1} + s_{11.1} + s_{12.1} + s_{13.1}$$
$$+ s_{72} + s_{82} + s_{92} + s_{10.2} + s_{11.2}$$
$$+ s_{63} + s_{73} + s_{83} + s_{93}$$
$$+ s_{44} + s_{64} + s_{74}$$
$$+ s_{15}.$$

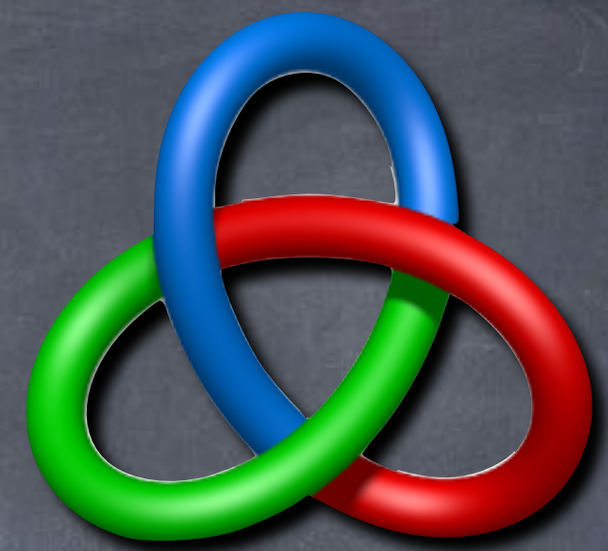
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# A Formula

$$A_{54321} = s_{11111} + s_{3111} + s_{4111} + s_{5111} + s_{6111}$$
$$+ s_{611} + s_{711} + 2s_{811} + s_{911} + s_{10.11}$$
$$+ s_{421} + s_{521} + s_{621} + s_{721} + s_{821}$$
$$+ s_{431} + s_{531} + s_{631} + s_{441}$$
$$+ s_{10.1} + s_{11.1} + s_{12.1} + s_{13.1}$$
$$+ s_{72} + s_{82} + s_{92} + s_{10.2} + s_{11.2}$$
$$+ s_{63} + s_{73} + s_{83} + s_{93}$$
$$+ s_{44} + s_{64} + s_{74}$$
$$+ s_{15}.$$

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$$\left(\sum_{k\geq 0} a^k e_k^{\perp} A_T\right)(q,t)$$



# POINCARÉ POLYNOMIAL OF THE TRIPLY GRADED KHOVANOV-ROZANSKY HOMOLOGY OF TORUS LINKS.

$e_k^{\perp}$ DUAL TO MULTIPLICATION BY ELEMENTARY

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# $(q,t)$-Evaluation

$$A_{321} = \Lambda_6 + \Lambda_{41} + \Lambda_{31} + \Lambda_{111}$$

$$a\, e_1^{\perp} A_{321} = a\,(\Lambda_5 + \Lambda_4 + \Lambda_3 + \Lambda_{31} + \Lambda_{21} + \Lambda_{11})$$

$$a^2 e_2^{\perp} A_{321} = a^2(\Lambda_3 + \Lambda_2 + \Lambda_1)$$

$$a^3 e_3^{\perp} A_{321} = a^3\, \Lambda_0$$

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$$a^3+a^2(q^3+q^2t+qt^2+t^3 \\ +q^2+qt+t^2+q+t)$$



$$+a(q^5+q^4t+q^3t^2+q^2t^3+qt^4+t^5 \\ +q^4+2q^3t+2q^2t^2+2qt^3+t^4 \\ +q^3+2q^2t+2qt^2+t^3+qt)$$

$$+(q^6+q^5t+q^4t^2+q^3t^3+q^2t^4+qt^5+t^6 \\ +q^4t+q^3t^2+q^2t^3+qt^4 \\ +q^3t+q^2t^2+qt^3)$$

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# DIAGONAL HARMONIC POLYNOMIALS

COMMUTING VARIABLES

BOSONS

$\omega$ INVOLUTION

ANTICOMMUTING VARIABLES

FERMIONS

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$$A_{321} = s_6 + s_{41} + s_{31} + s_{111}$$

$\omega$

$$s_{111111} + s_{2111} + s_{211} + s_3$$

# Triangular Parking Functions

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# Enumération

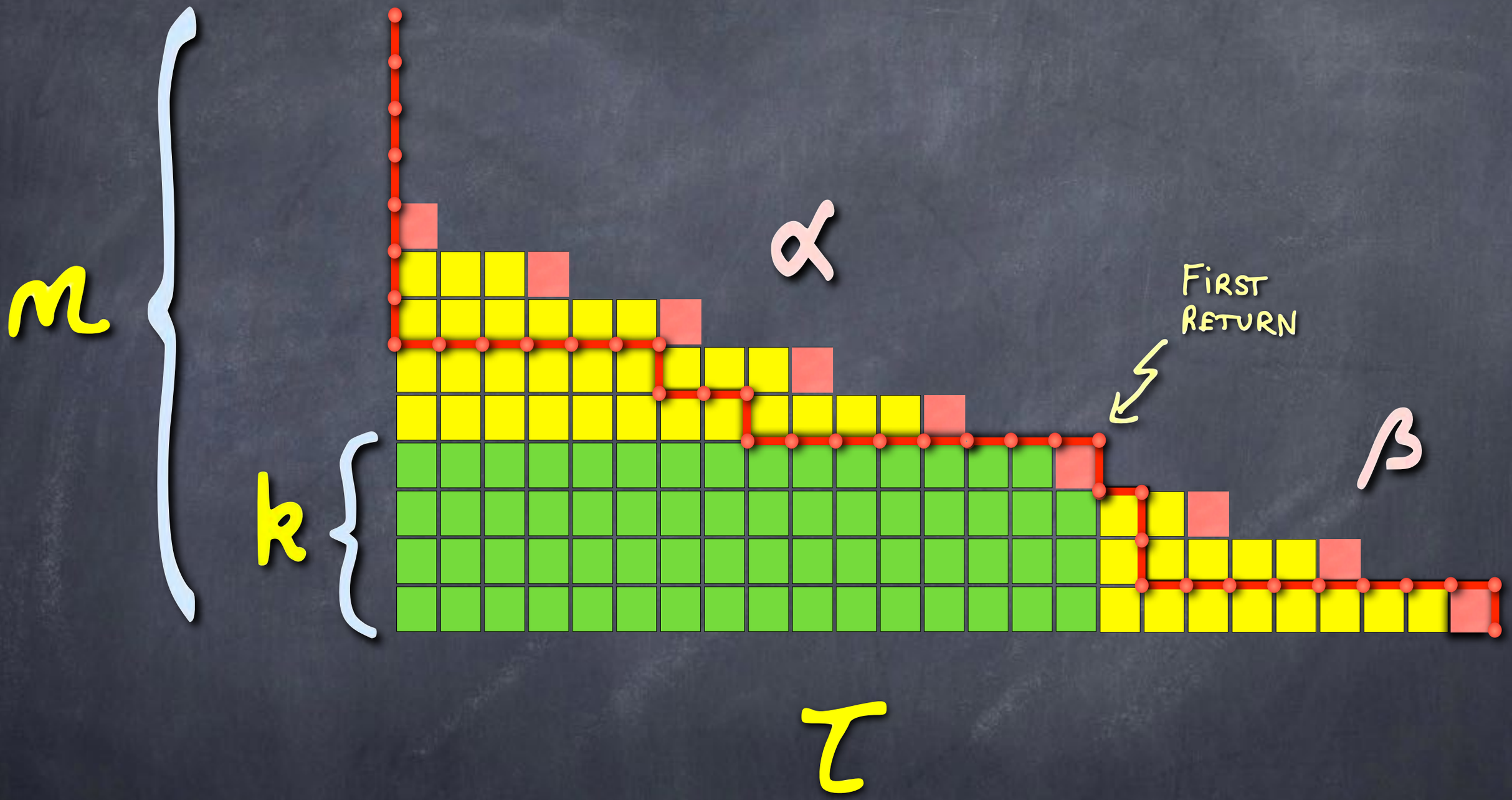
$$\rho_\tau^m(X) := \sum_{\alpha \subseteq \tau} \delta_{(\alpha + 1^m)/\alpha}(X)$$

## Proposition

$$\rho_\tau^m(X) = \sum_{(\alpha,\beta)} \rho_\alpha^{m-k}(X)\rho_\beta^k(X)$$

$$\rho_\varepsilon^m(X) = e_m(X)$$

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$$\rho_\tau^m(f;X) := \sum_{\alpha\subseteq\tau} f^{|\tau|-|\alpha|}\, \Delta_{(\alpha+1^m)/\alpha}(X)$$

$$\rho_\tau^m(f,t;X) := \sum_{\alpha\subseteq\tau} f^{|\tau|-|\alpha|}\, \mathbb{L}_\alpha^m(t;X)$$

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$$P_\tau^m(q,t;X) := \sum_{\alpha \subseteq \tau} q^{|\tau|-|\alpha|} \, \mathbb{L}_\alpha^m(t;X)$$

**THEOREM** (BLASIAK HAIMAN MORSE PUN SEELINGER)

$$P_\tau^m(q,t;X) = D_{b_1 b_2 \cdots b_m} \cdot 1$$

CALCULATIONS IN THE ELLIPTIC HALL ALGEBRA.

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# NEGUT FORMULA

$$\mathcal{P}^{n}_{\Pi}\left(\left(\frac{f}{t_f}, t_f^{i}\right); x\right) = \sum_{\mu \vdash n} \left( \sum_{\theta \in SYT(\mu)} z_{\tau}(\theta)\, \Omega(\theta) \right) \tilde{H}_{\mu}\left(\frac{f}{t_f}, t_f^{i}; x\right)$$

MODIFIED MACDONALD POLYNOMIALS

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# SCHUR POSITIVITY CONJECTURE

$$1 \otimes s_4 + (s_1 + s_2 + s_3) \otimes s_{31}$$
$$+ (s_{21} + s_2 + s_4) \otimes s_{22}$$
$$+ (s_{11} + s_{21} + s_{31} + s_3 + s_4 + s_5) \otimes s_{211}$$
$$+ (\underbrace{s_{111} + s_{31} + s_{41} + s_6}) \otimes s_{1111}$$
$$A_\Gamma(q,t)$$

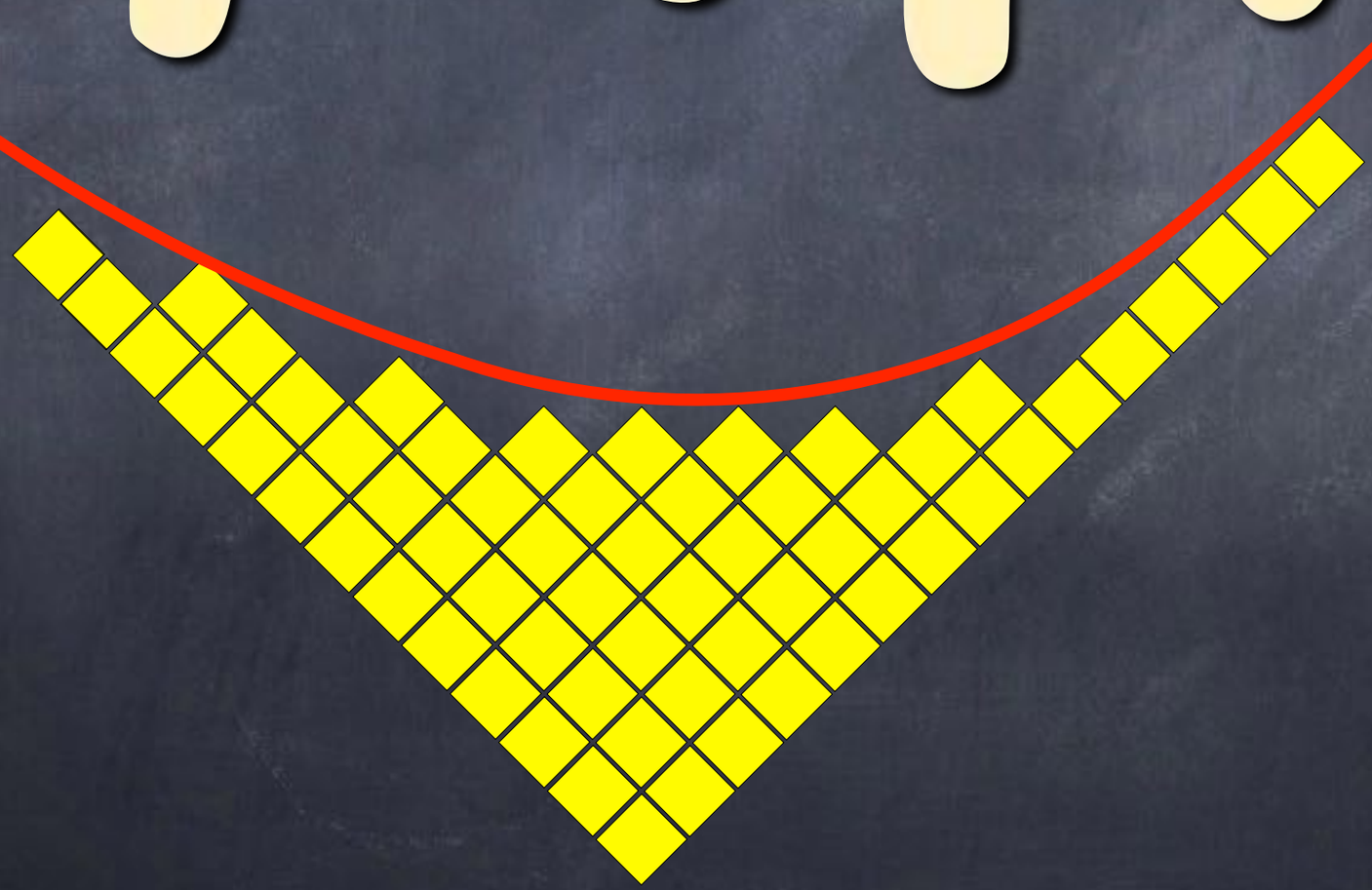
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- VARIATIONS ON DIAGONAL COINVARIANT MODULES
- BOSONIC-FERMIONIC ARXIV:2005.00924
- HILBERT SCHEME OF POINTS IN $\mathbb{C}^k$
- ...



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# FIN



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