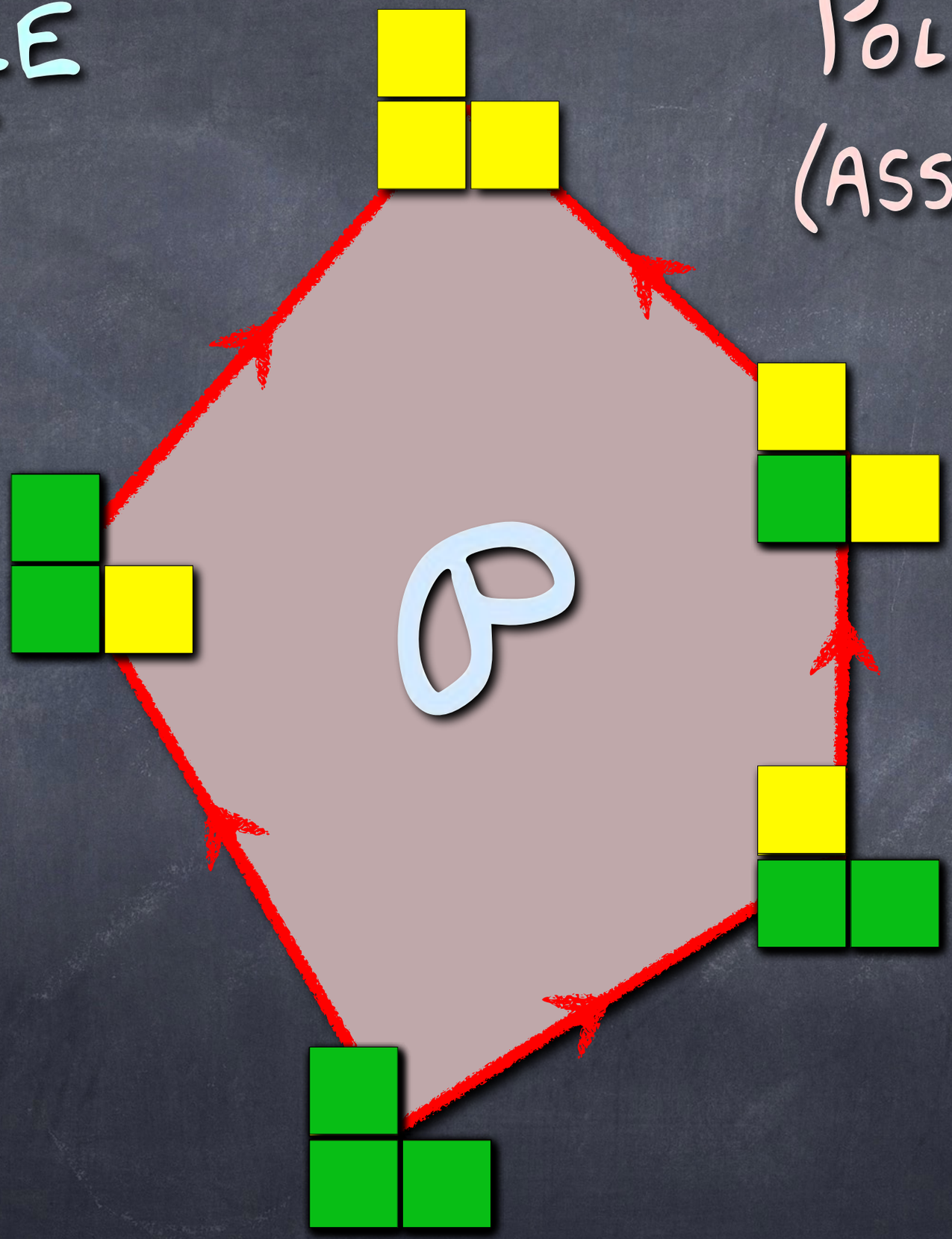
h-Analogue du h-Polynôme

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ENSEMBLE ORDONNÉ (POSET)

POLYTOPE (ASSOCIAÈDRE)

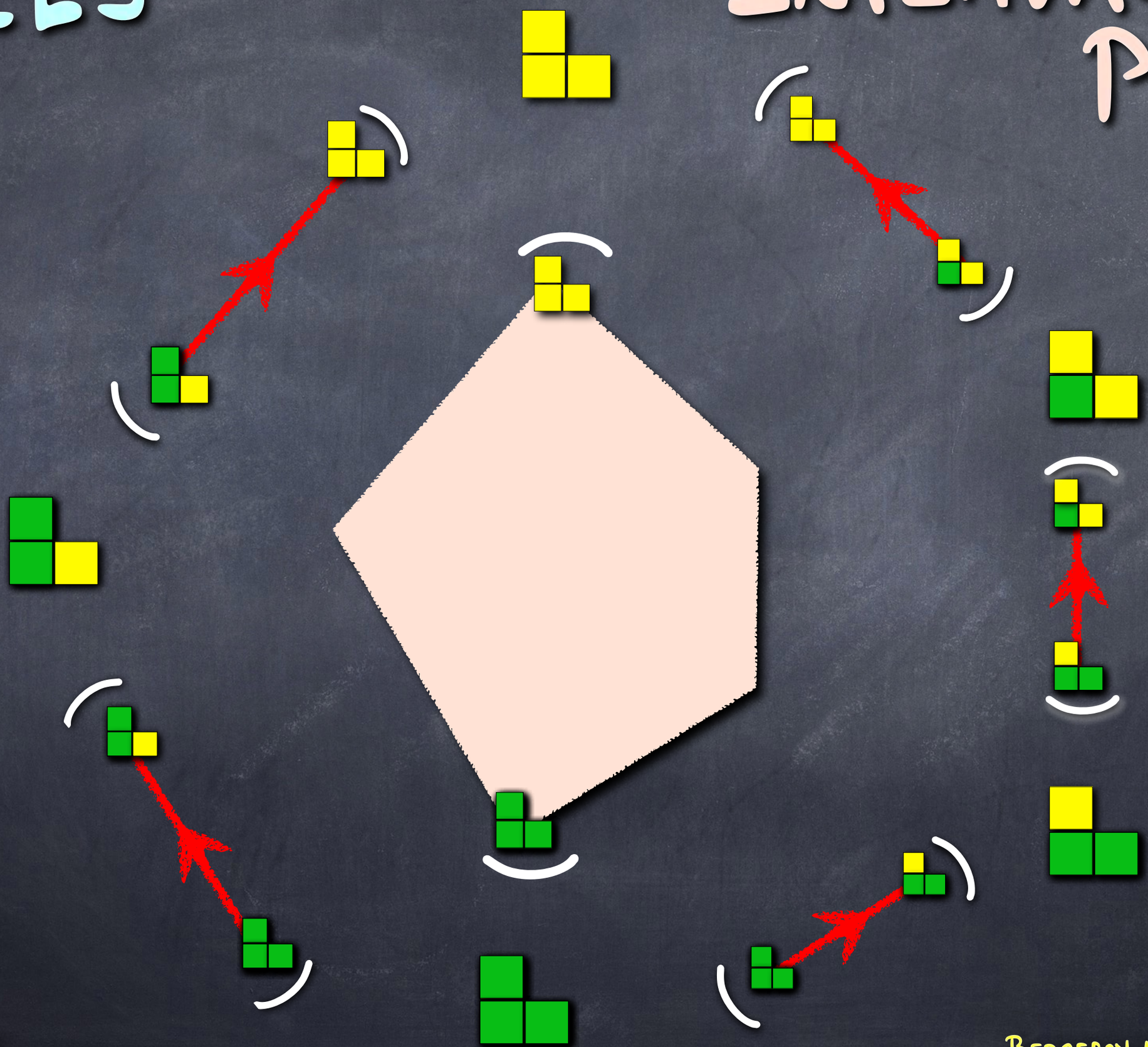


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FACES

INTERVALLES PURS



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FACES = INTERVALLES PURS



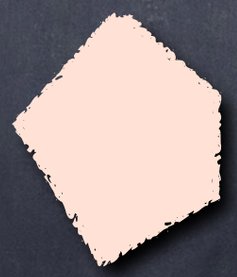
5





5

( , )



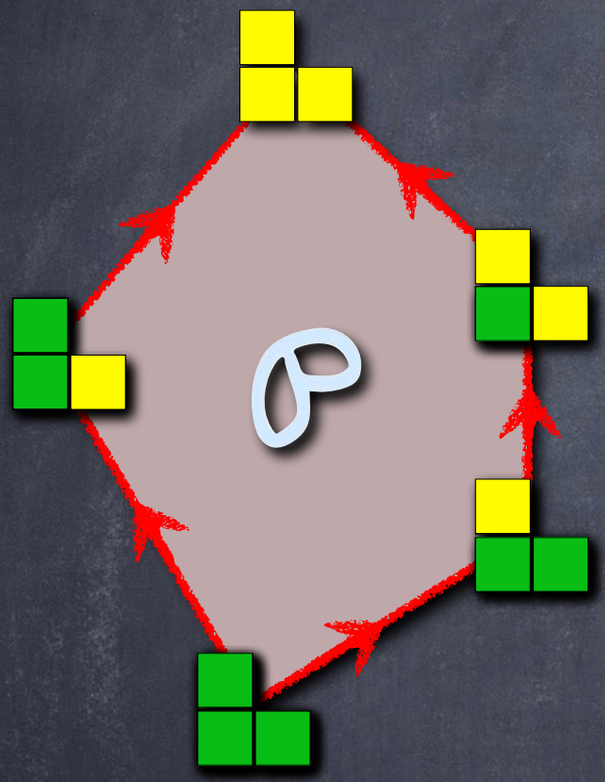
1

( , )

VECTEUR FACES $f_\rho := (5, 5, 1)$

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Vecteur Faces $f_\rho := (5,5,1)$



Polynôme Faces

$$f_\rho(x) := 5 + 5x + 1x^2$$

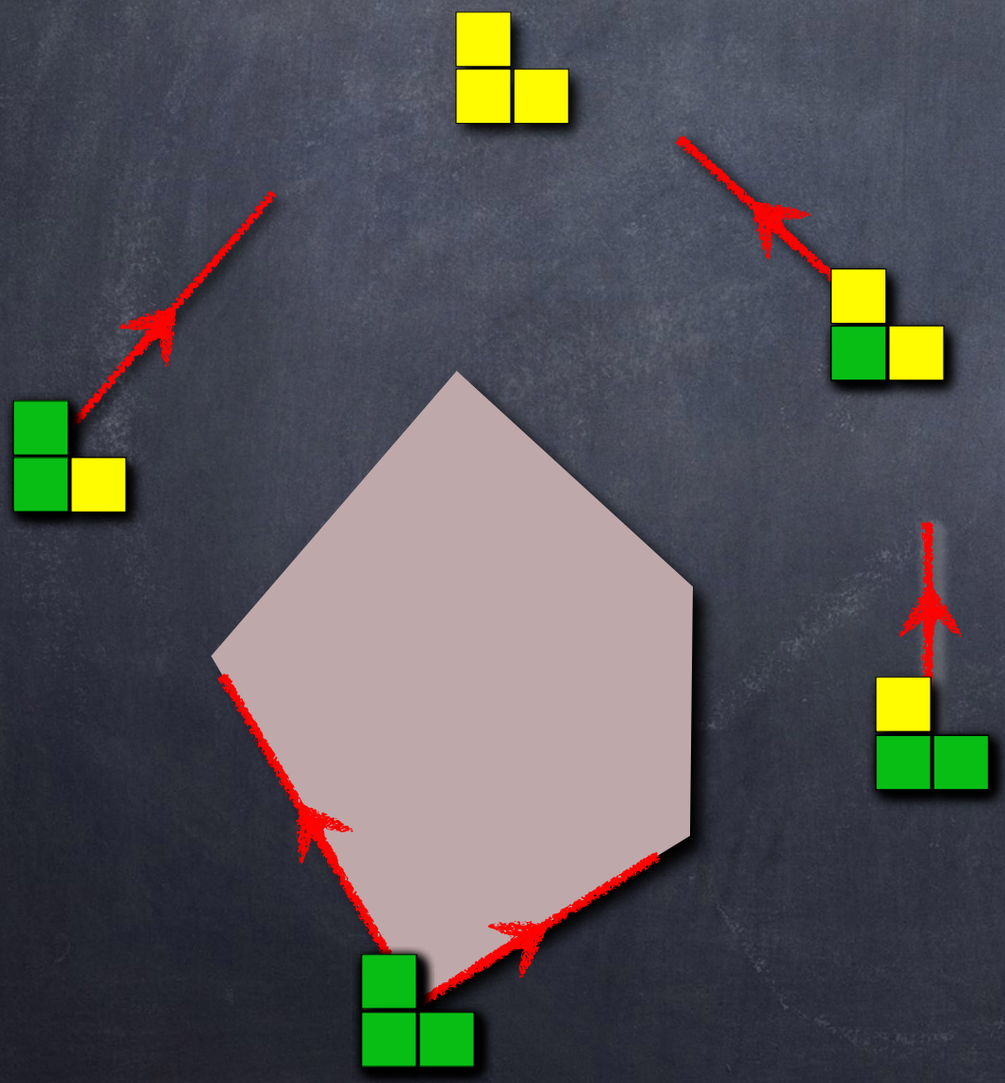
$$h_\rho(x) := f_\rho(x-1)$$

$$= 1 + 3x + 1x^2$$

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h-Polynôme

$$h_{\mathcal{P}}(x) = 1 + 3x + 1x^2$$



"Half-open" faces

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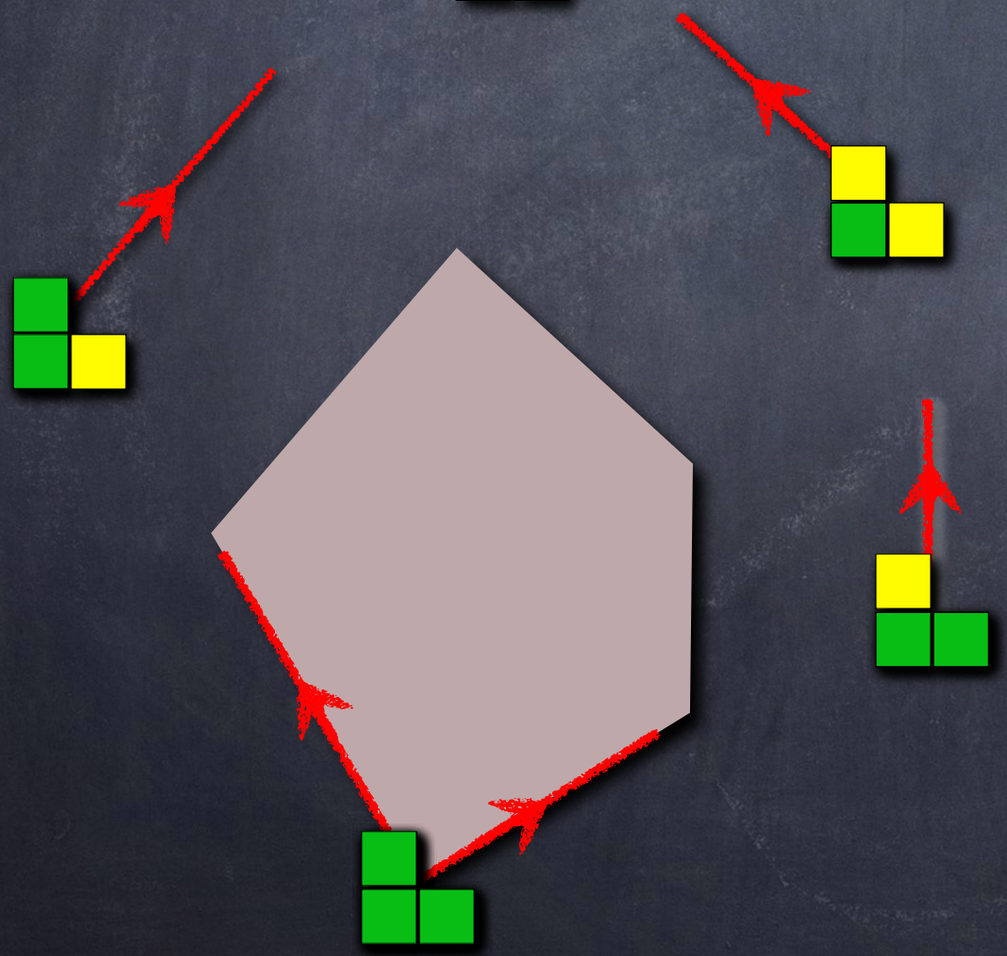
h-Polynôme

$$h_{\partial\sigma}(x) = 1 + 3x + 1x^2$$

1 Sommet







3 Demi arête



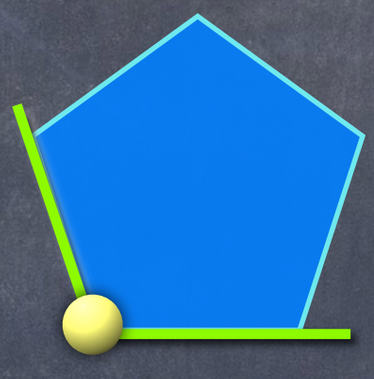
1 Demi face

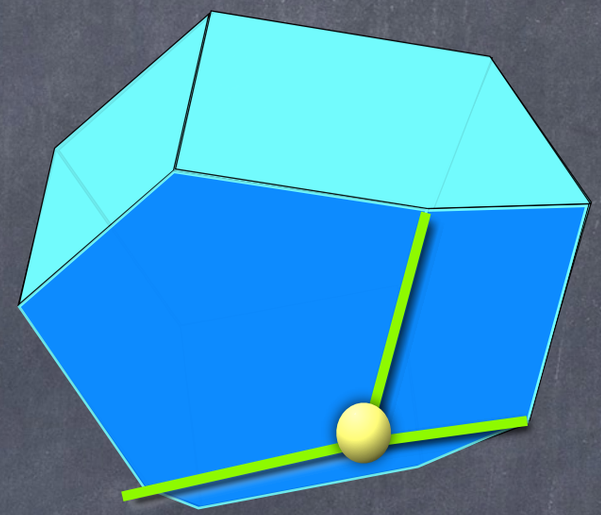


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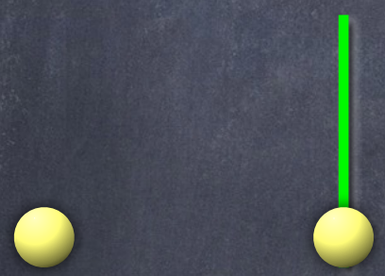
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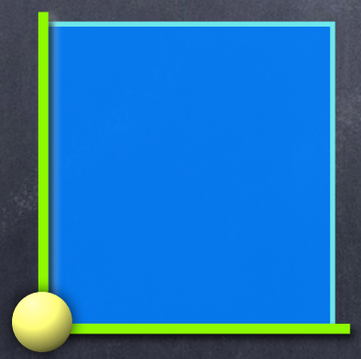
Half-open faces

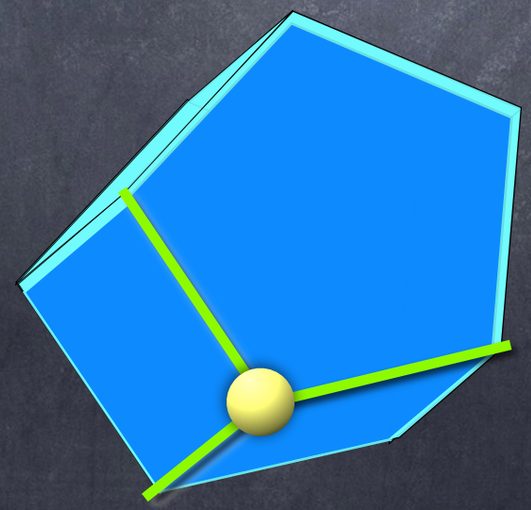




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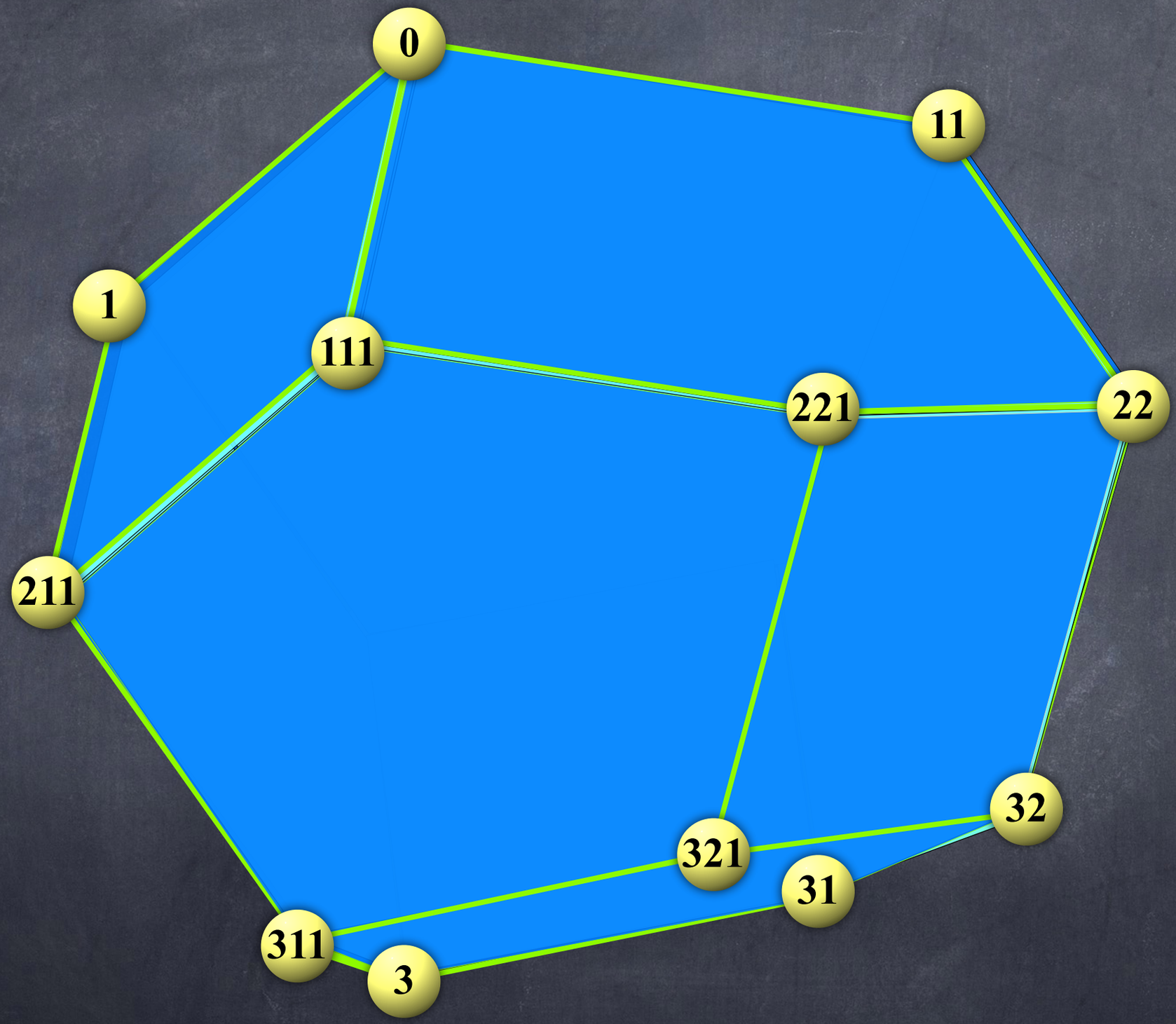




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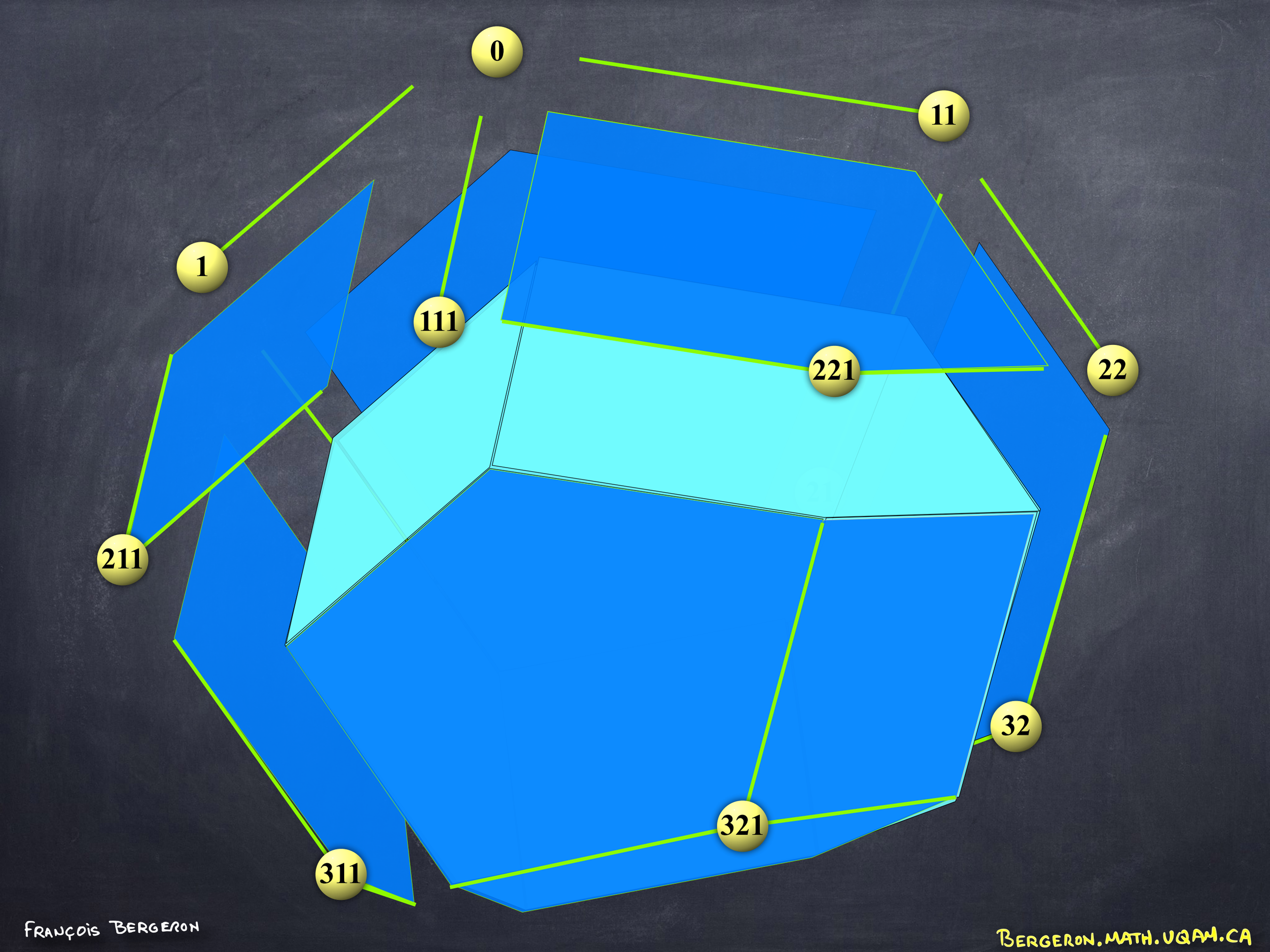
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POLYTOPE ORDONNÉ (TREILLIS)

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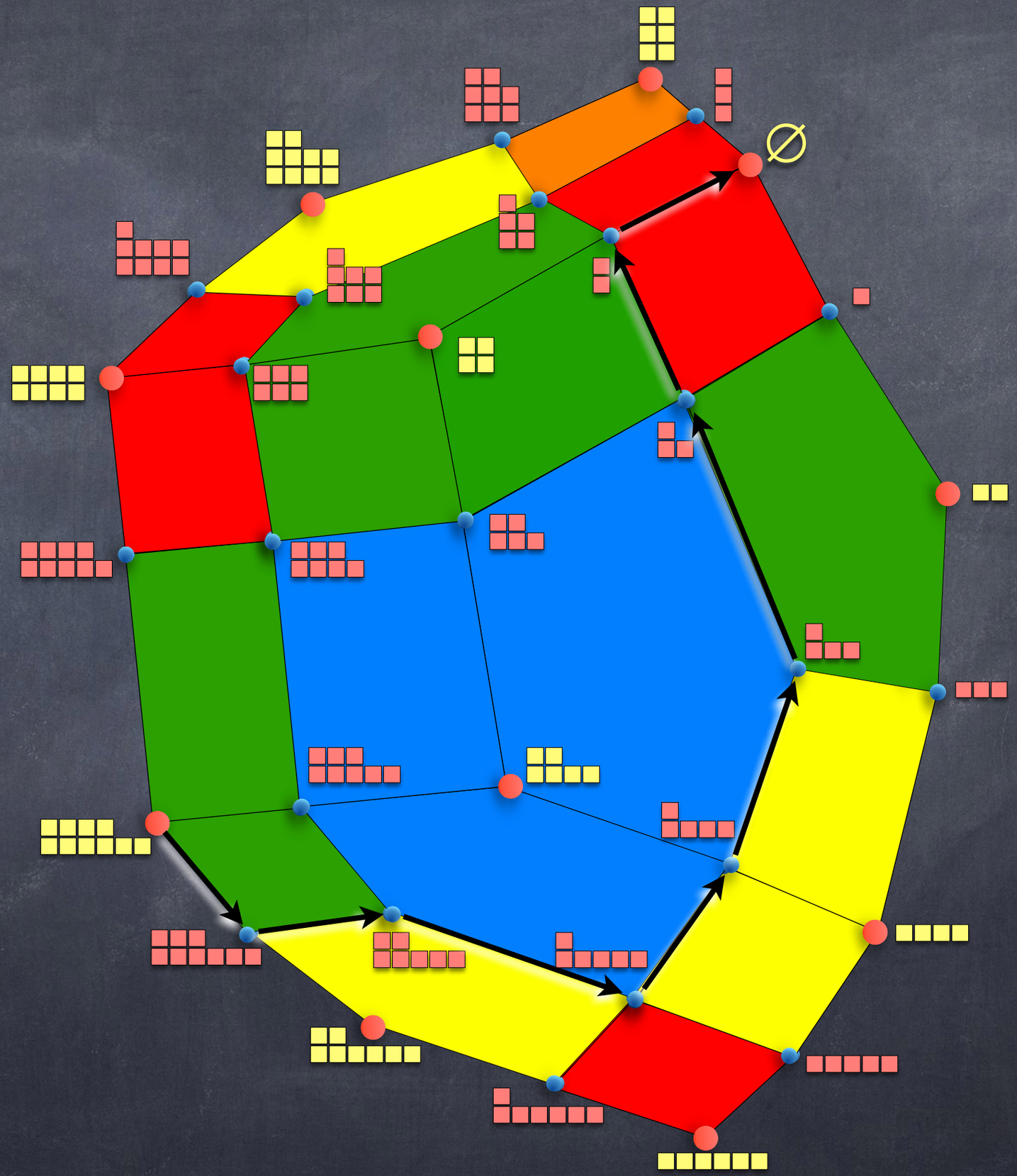
POLYTOPE ORDONNÉ (TREILLIS)

$\alpha \leqslant \beta$

ORDRE

$\alpha \to \beta$

COUVERTURE



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$\mathcal{P}$ POLYTOPE ORDONNÉ (TREILLIS)

$$\alpha \in \mathcal{P}$$

$$S_\alpha := \{\beta \mid \alpha \to \beta\}$$

$$T \subseteq S_\alpha$$

FACES $\quad F_T := (\alpha, \bigvee_{\beta \in T} \beta)$

H-FACES $\quad H_\alpha := [\alpha, \bigvee_{\alpha \to \beta} \beta)$

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$$f_\rho(x) := \sum_{\alpha \in \rho} \sum_{T \subseteq S_\alpha} x^{\dim(F_T)}$$

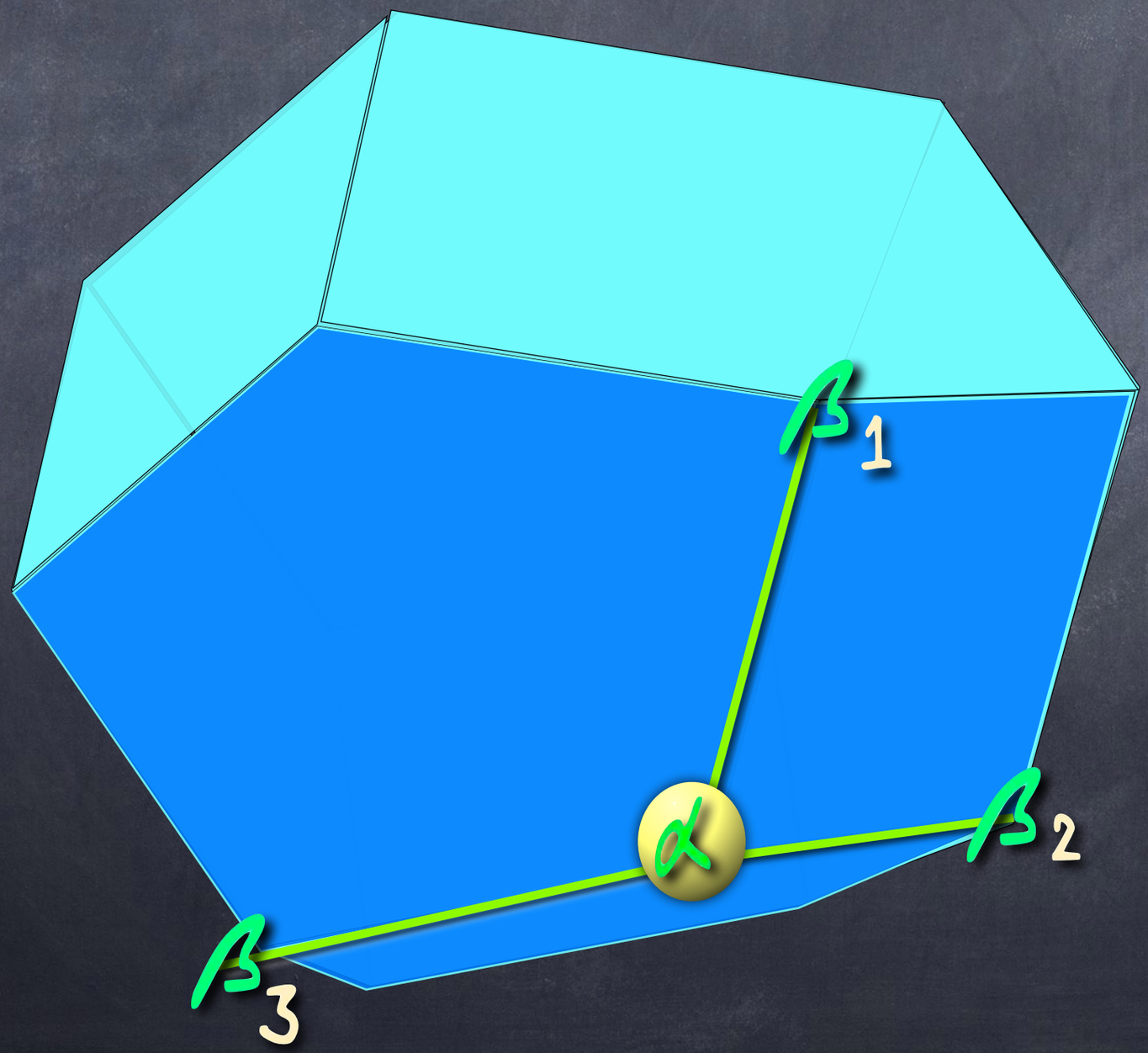
$$h_\rho(x) := \sum_{\alpha \in \rho} x^{\dim(H_\alpha)}$$

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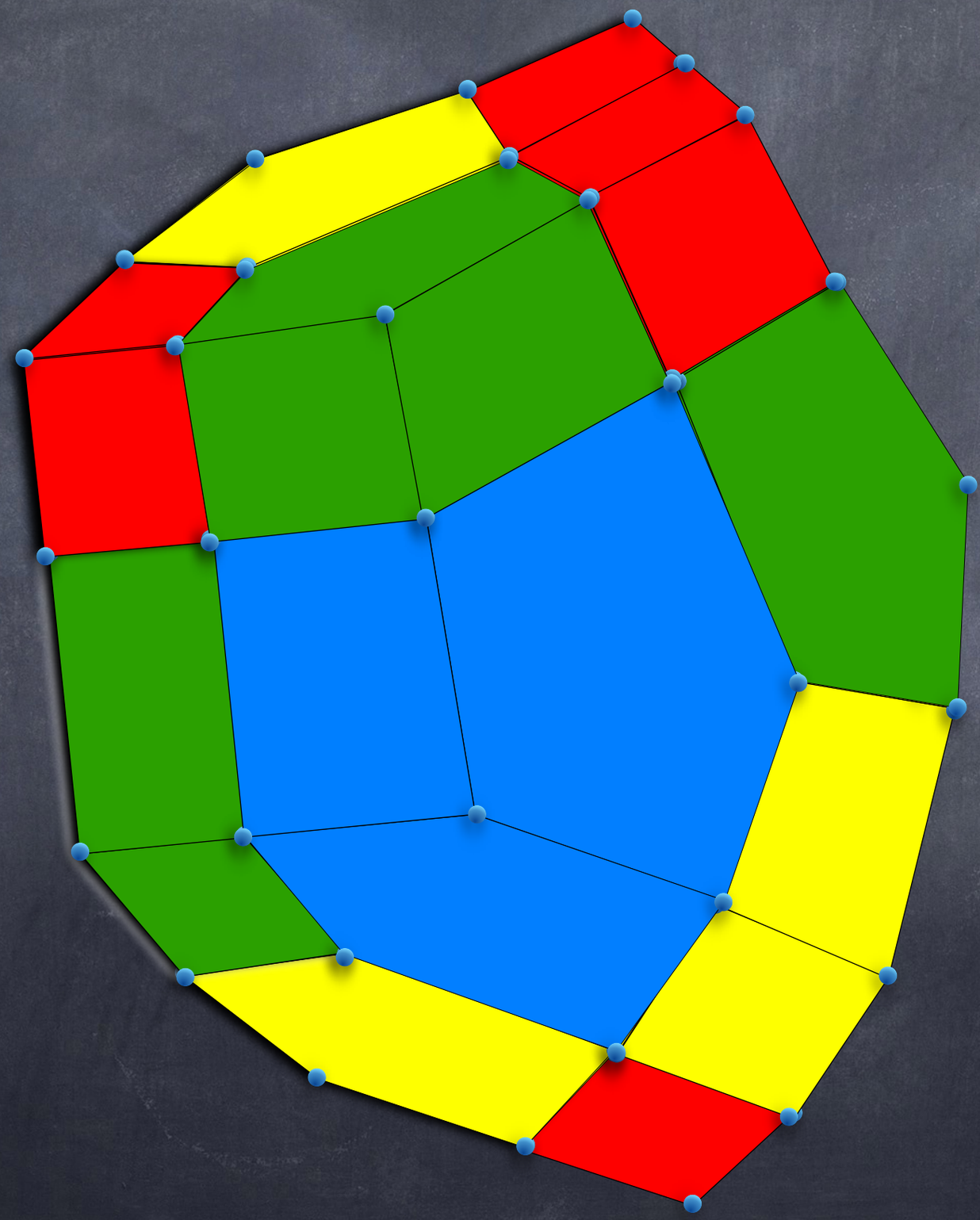
$$h_\rho(x) := \sum_{\alpha \in \rho} x^{\mathrm{CARD}(S_\alpha)}$$

$$S_\alpha := \{\beta \mid \alpha \to \beta\}$$



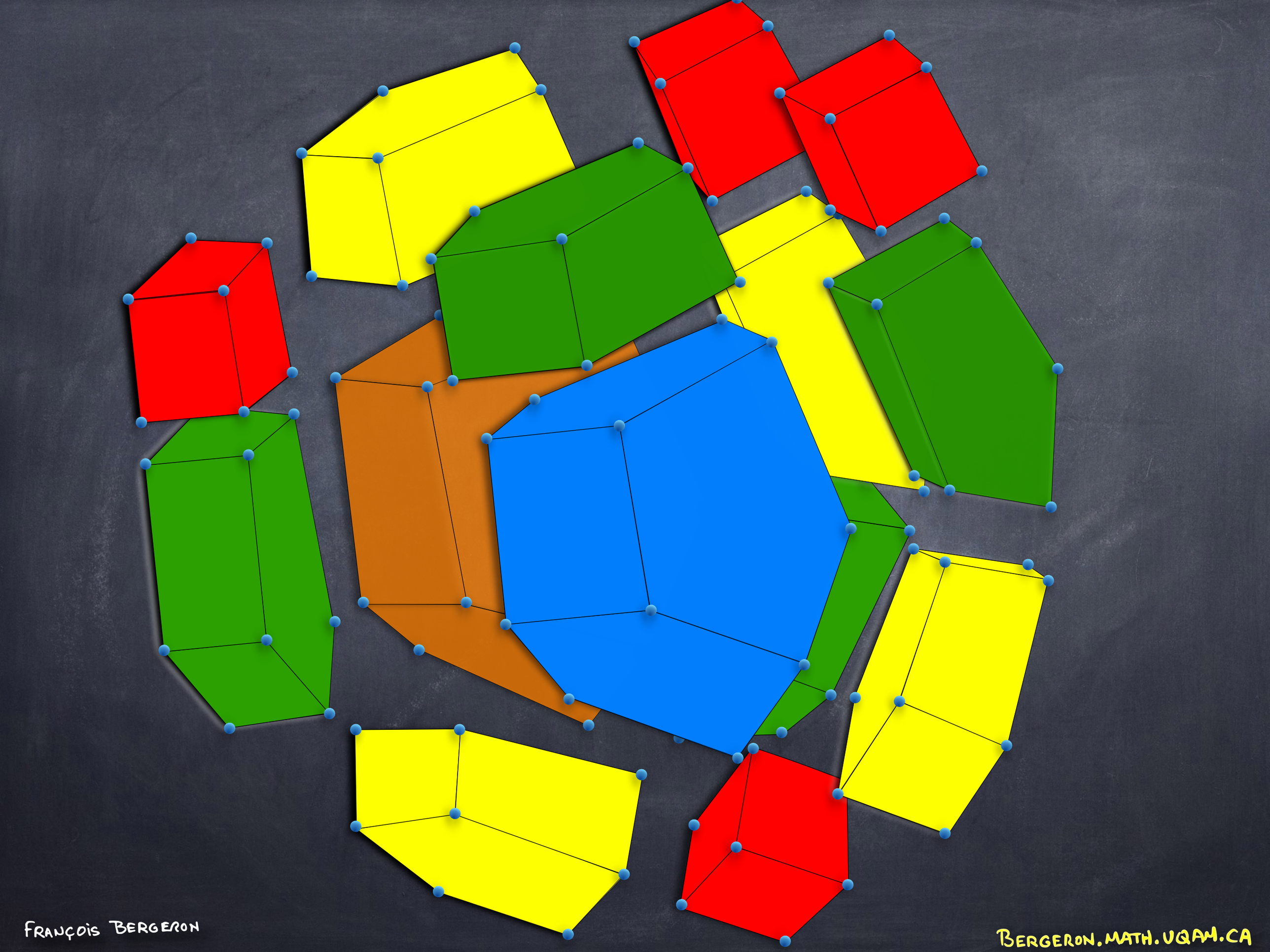
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COMPLEXES ASSOCIAÈDRAUX



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COMPLEXES ASSOCIAÈDRAUX

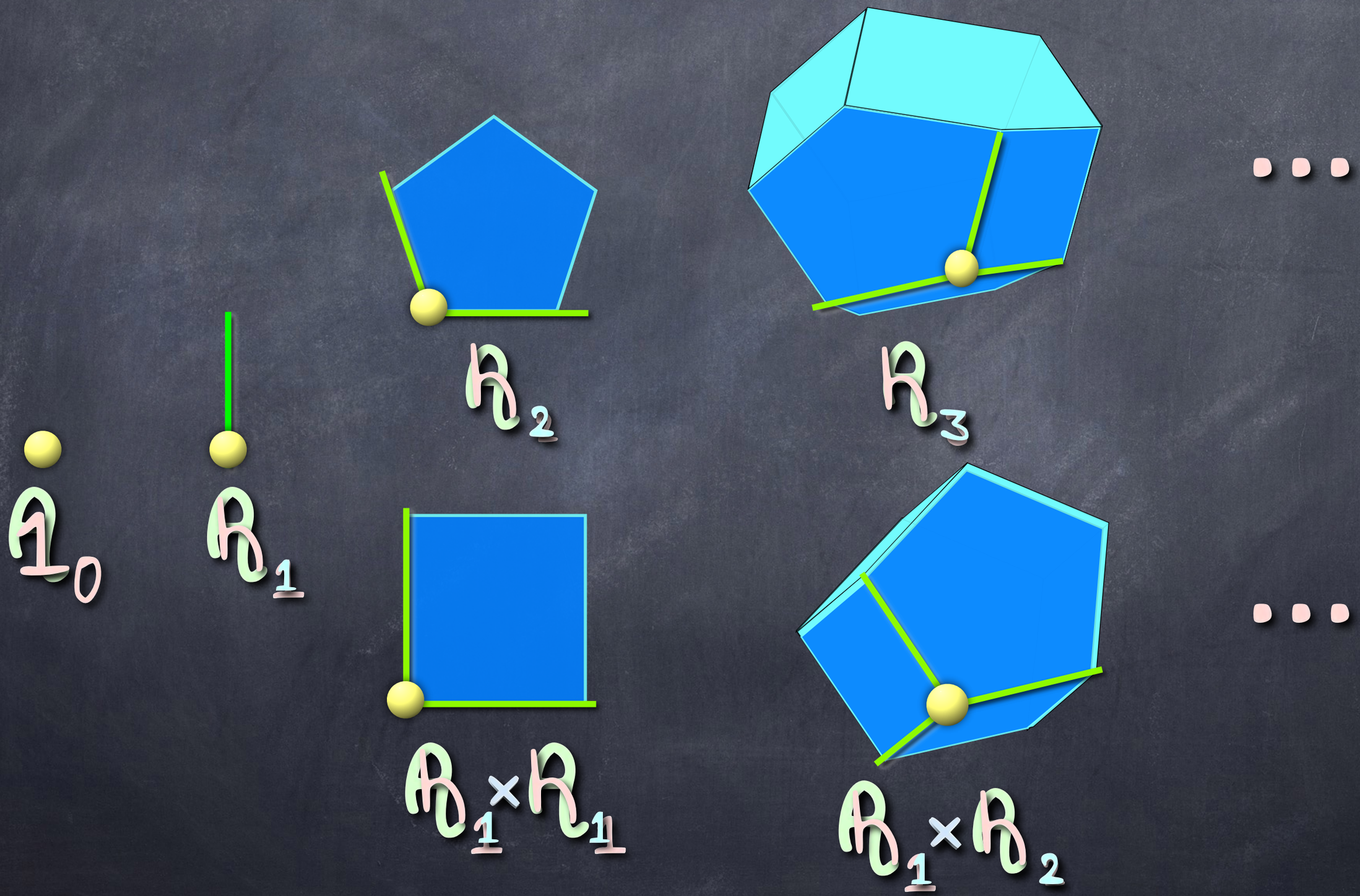
- PURE: FACES MAXIMALES TOUTES DE DIMENSION d
- FACES MAXIMALES RECOLLÉES LE LONG DE FACES DE DIMENSION $d-1$
- CONVEXE
- FACES PRODUITS D'ASSOCIAÈDRES

$$A_\mu := A_{\mu_1} \times A_{\mu_2} \times \cdots \times A_{\mu_k}$$

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CODAGE DES H-FACES $A_\mu \rightsquigarrow h_\mu$



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CODAGE DES H-FACES H_α

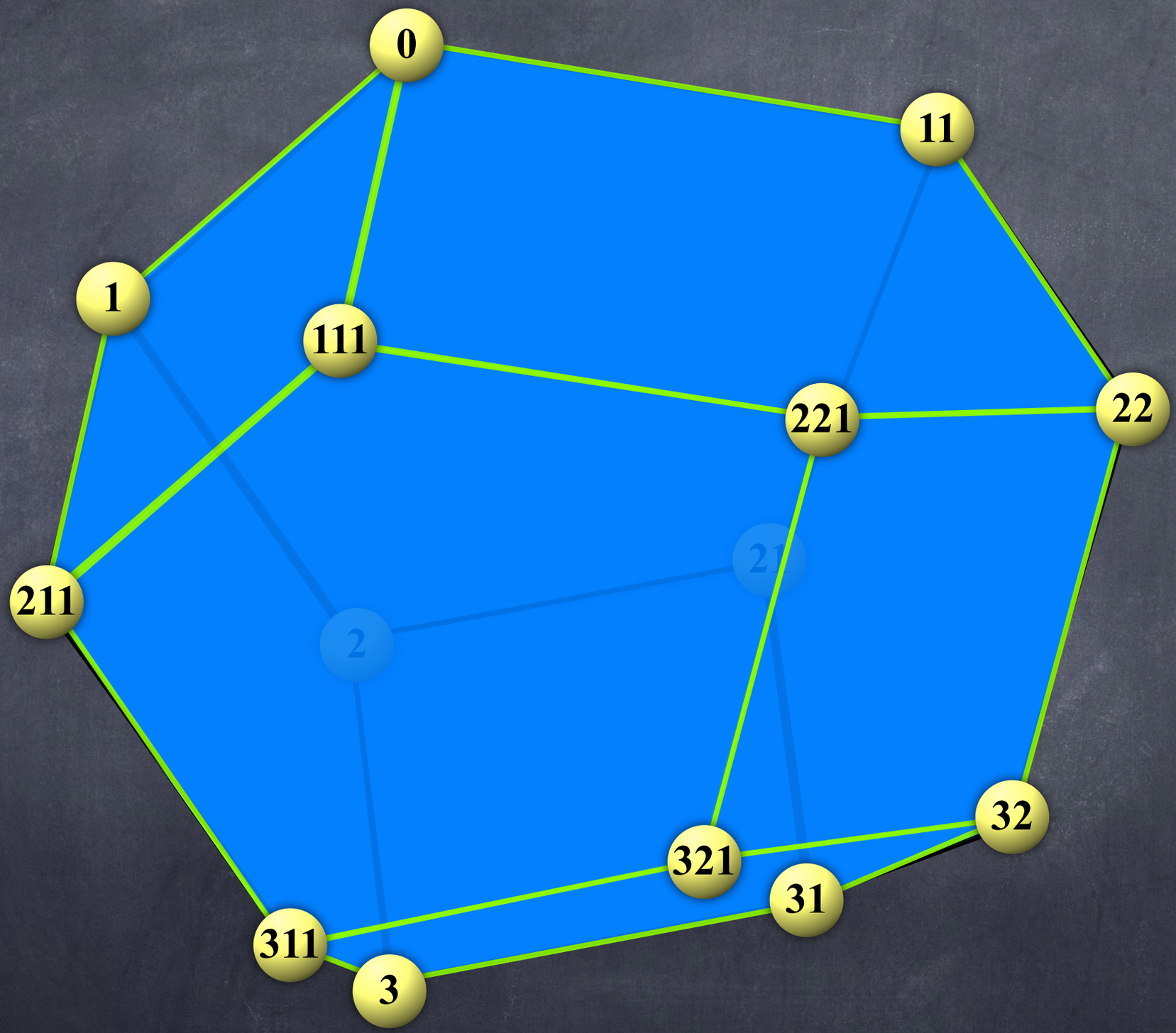
$$H_\alpha \rightsquigarrow \left[\alpha, \bigvee_{\alpha\to\beta} \beta\right] \cong A_\mu$$

$$h_\mu, \quad \mu \vdash \dim(H_\alpha) = \mathrm{CARD}(S_\alpha)$$

$\left[\alpha, \bigvee_{\alpha\to\beta} \beta\right]$ # VERTICES EQUAL TO $\prod_i \mathrm{CAT}_{(\mu_i+1)}$

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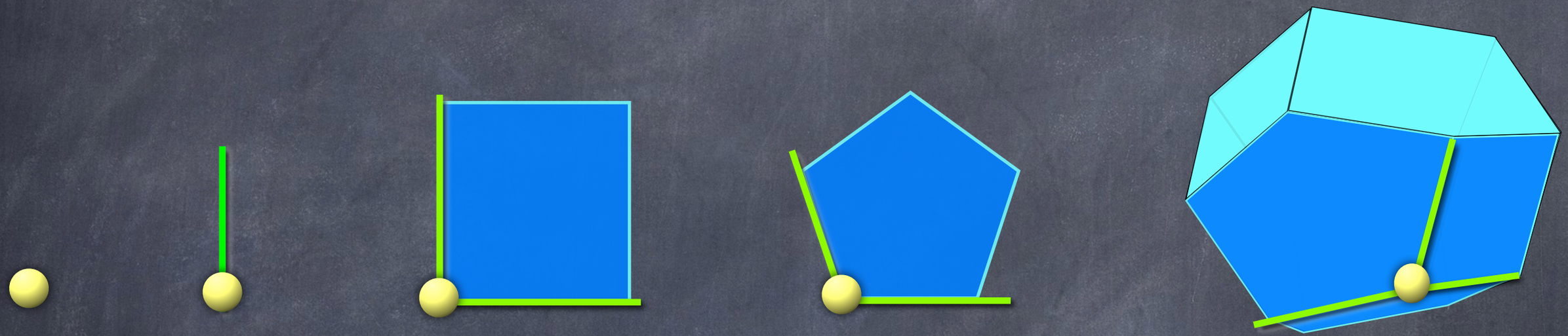
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ENUMERATION

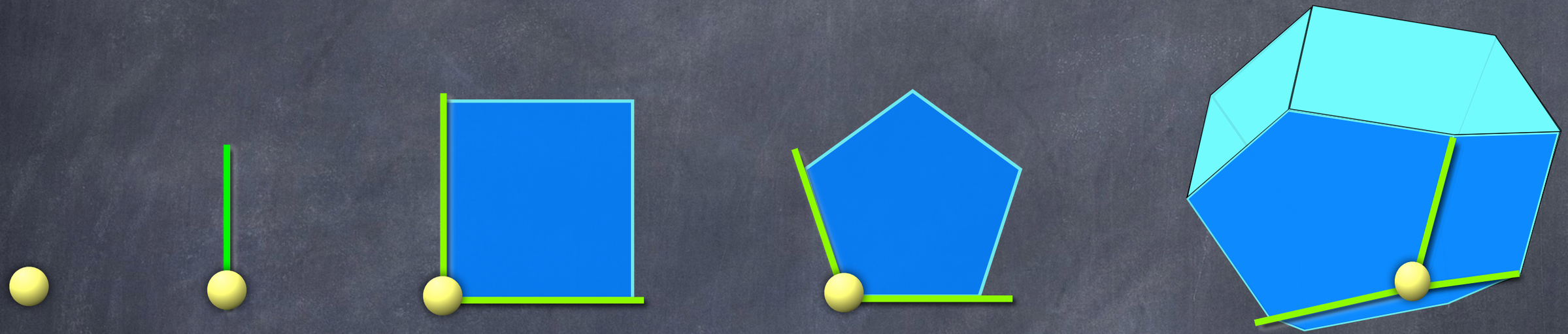


1 6 2 4 1

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h-Enumeration

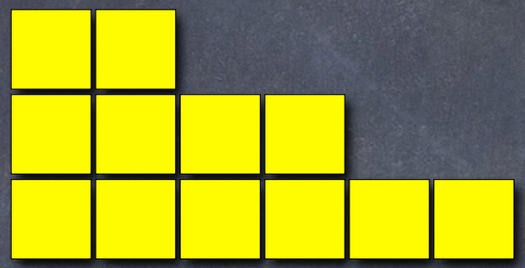


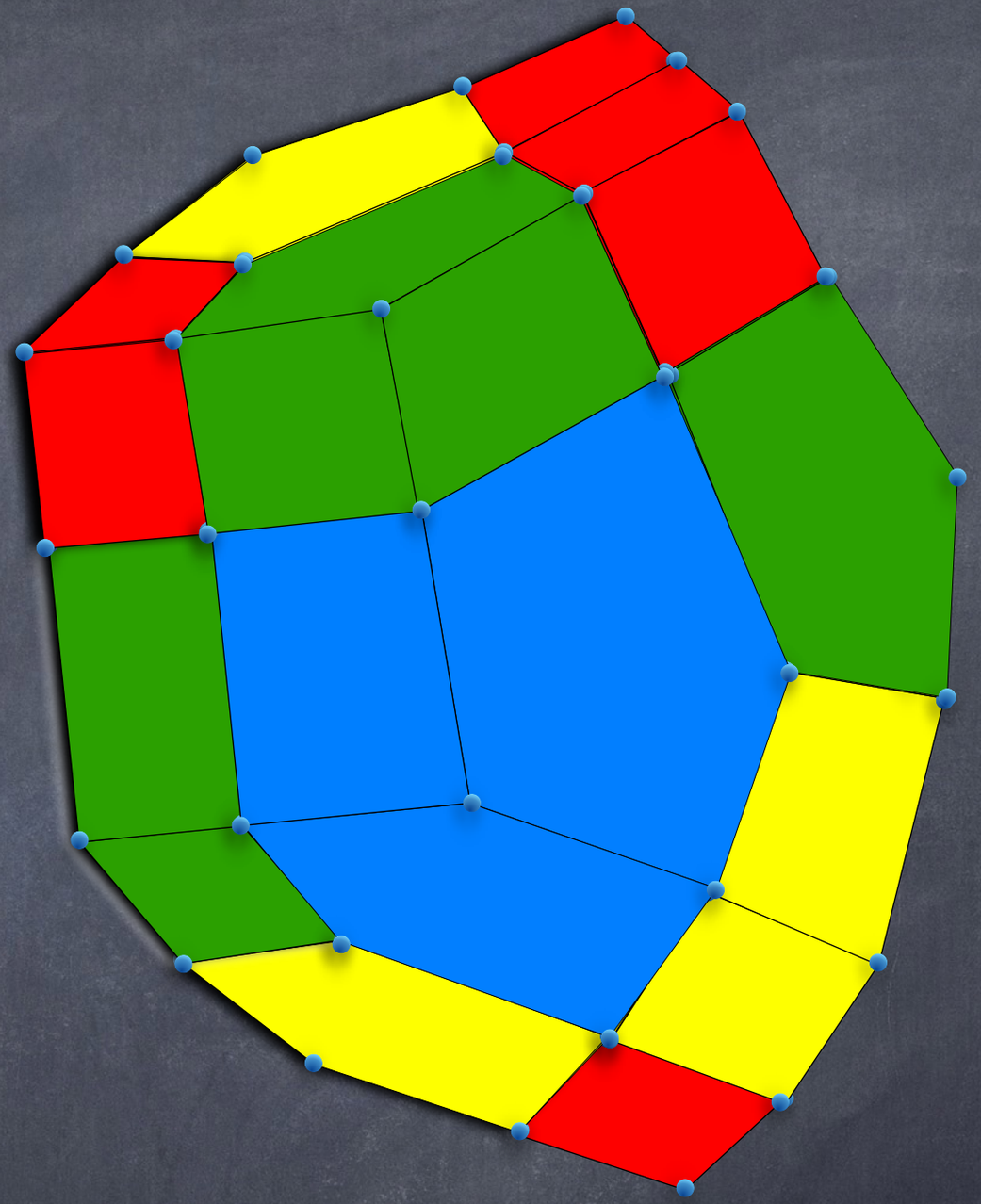
$$1 + 6\,h_1 + 2\,h_1 h_1 + 4\,h_2 + h_3$$

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τ-TAMARI

$\tau =$ 

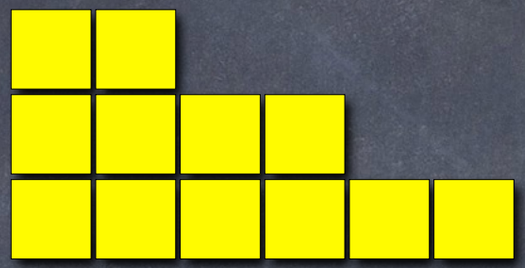


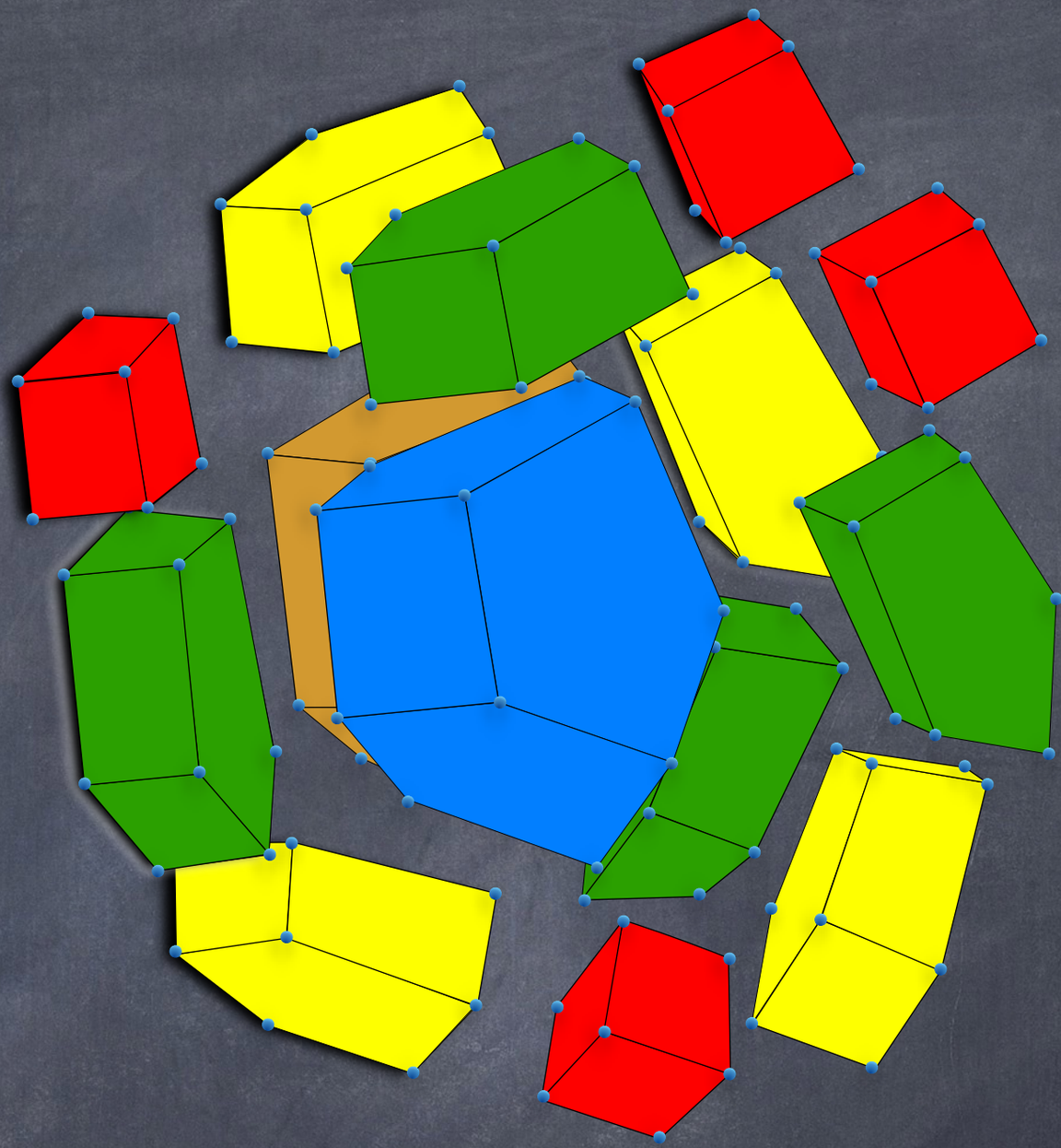
$$1 + 12\,h_1 + 20\,h_{11} + 8\,h_2 + 4\,h_{111} + 8\,h_{21} + 2\,h_3$$

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τ-TAMARI

$\tau =$ 



$$1 + 12\,h_1 + 20\,h_{11} + 8\,h_2 + \underbrace{4\,h_{111} + 8\,h_{21} + 2\,h_3}_{14}$$

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$\mathcal{H}_\sigma$

hh-Polynôme

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$$\mathcal{H}_{\mathcal{P}} := \sum_{\alpha \in \mathcal{P}} h\left(\left[\alpha, \bigvee_{\alpha \to \beta} \beta\right]\right)$$

$$h\left(\left[\alpha, \bigvee_{\alpha \to \beta} \beta\right]\right) := h_{\mu}$$

$$= h_{\mu_1} h_{\mu_2} \cdots h_{\mu_\ell}$$

$$\left[\alpha, \bigvee_{\alpha \to \beta} \beta\right] \simeq A_{\mu_1} \times A_{\mu_2} \times \cdots \times A_{\mu_\ell}$$

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$$\left[\alpha, \bigvee_{\alpha \to \beta} \beta\right] \cong A_{\mu_1} \times A_{\mu_2} \times \cdots \times A_{\mu_\ell}$$

$$\mu \vdash \text{PARTAGE DE CARD}(S_\alpha)$$

$$\#\left[\alpha, \bigvee_{\alpha \to \beta} \beta\right] = C_{(\mu_1+1)}\, C_{(\mu_2+1)} \cdots C_{(\mu_\ell+1)}$$

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Smalest Example of

$$\mu \neq \nu \qquad |\mu| = |\nu|$$

$$C_{(\mu_1+1)}\, C_{(\mu_2+1)} \cdots C_{(\mu_\ell+1)} = C_{(\nu_1+1)}\, C_{(\nu_2+1)} \cdots C_{(\nu_k+1)}$$

$$\mu = (10,1,1) \qquad \nu = (9,3)$$

$$C_{(10+1)} C_{(1+1)} C_{(1+1)} = C_{(9+1)} C_{(3+1)} = 235144$$

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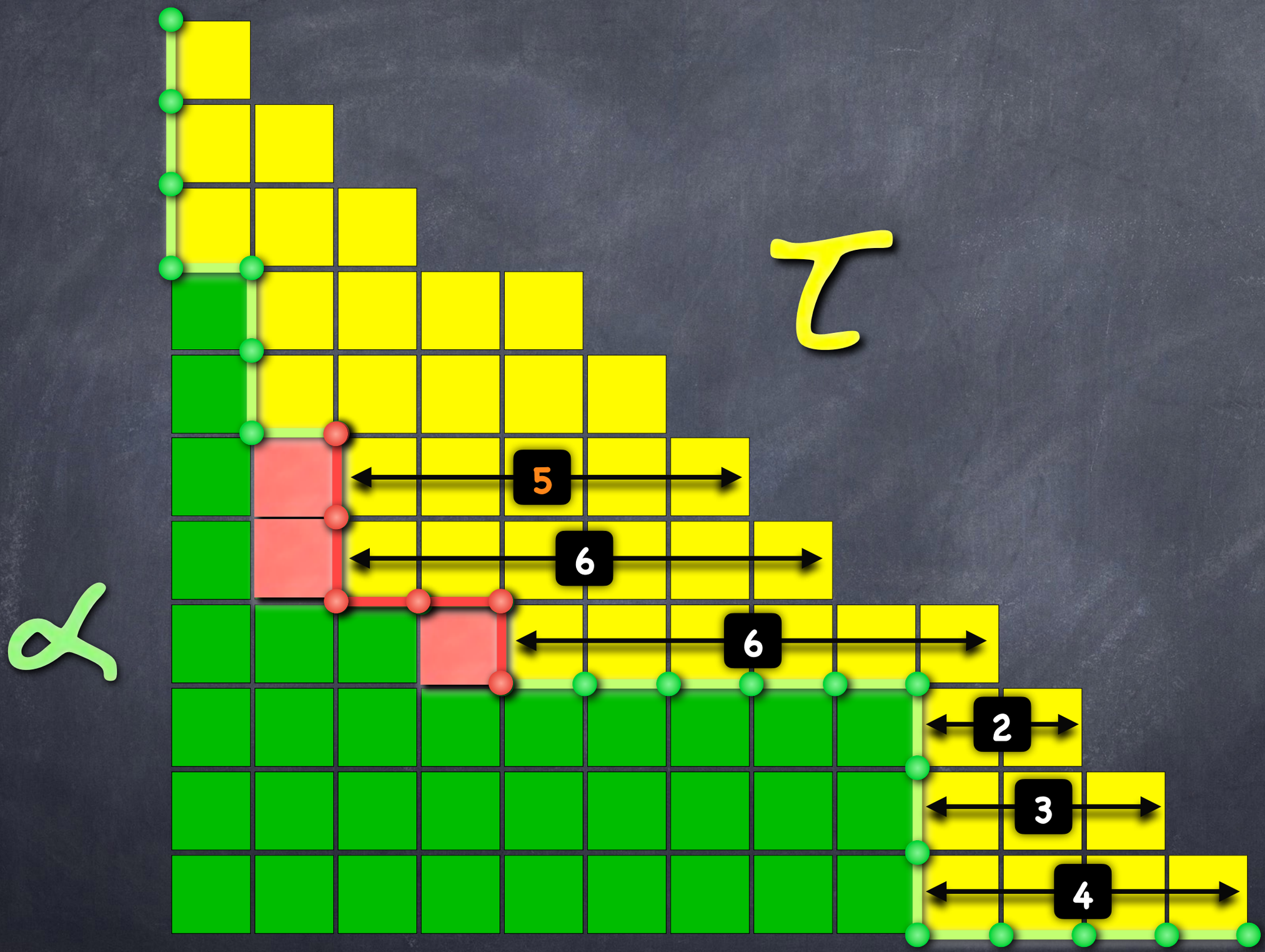
TREILLIS
T-TAMARI

PRÉVILLE-RATELLE, VIENNOT
AN EXTENSION OF
TAMARI LATTICES (2015)

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τ

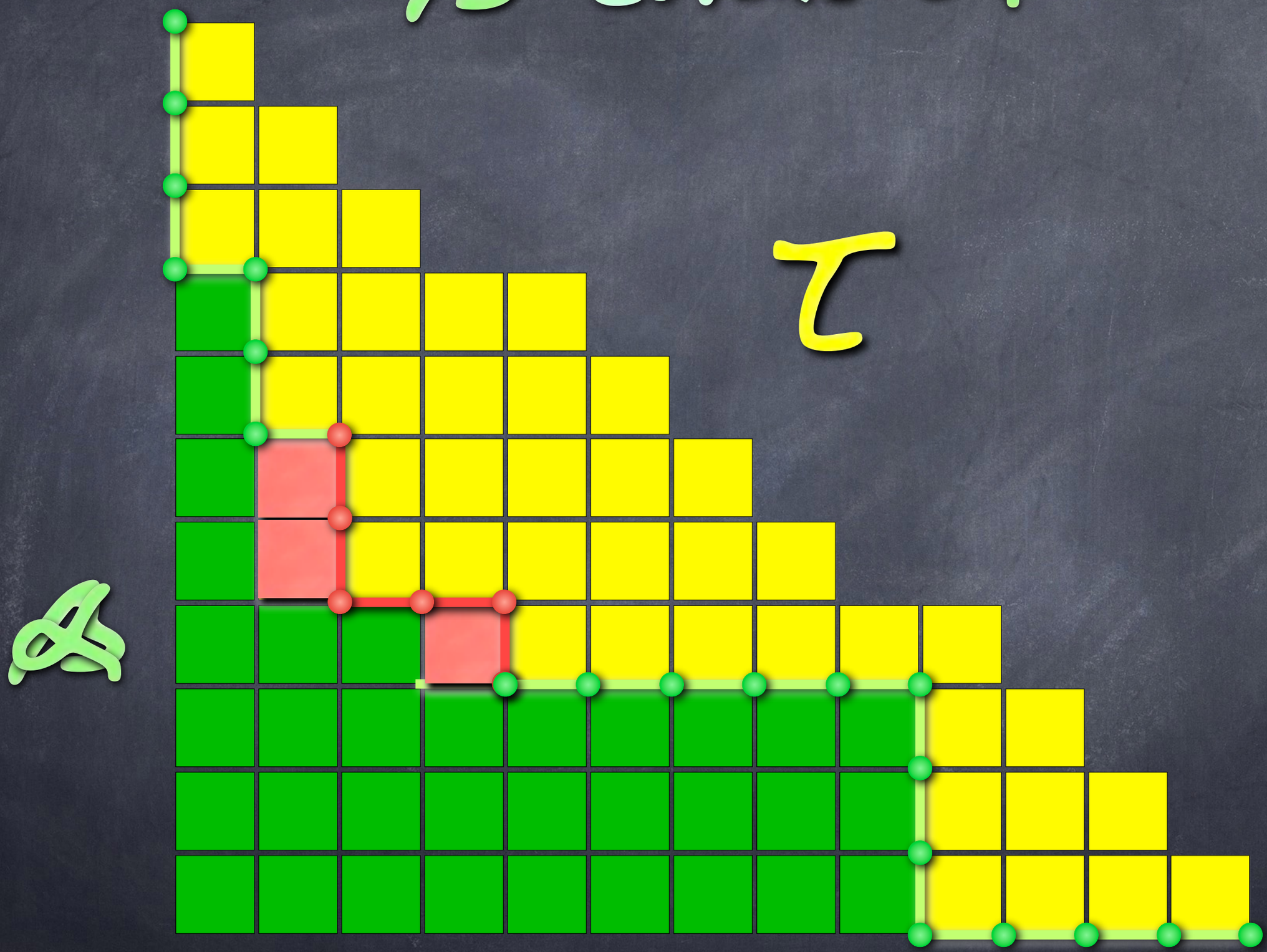


α

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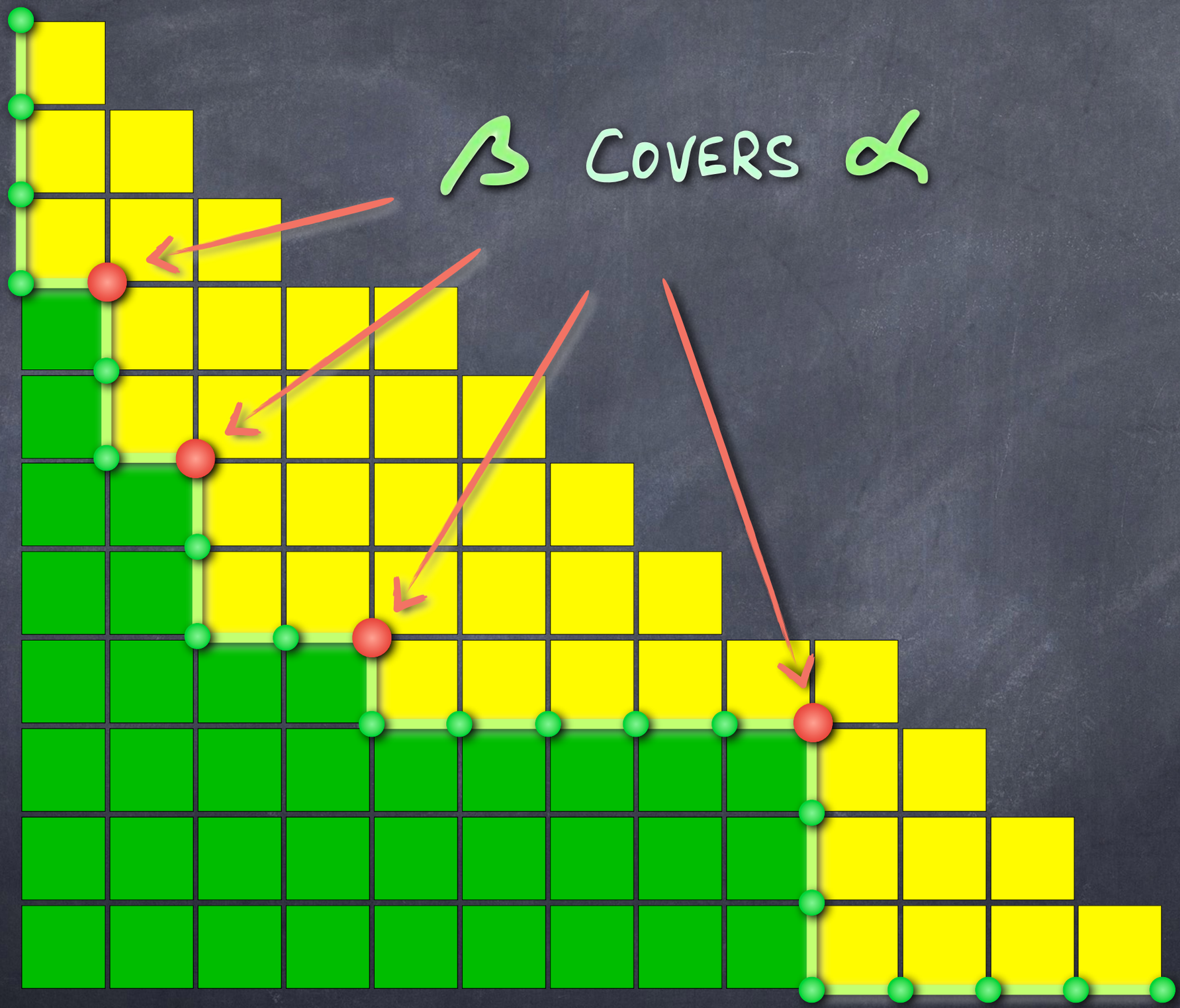
β Covers α

τ



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Proposition

$$\tau = (dm, d(m-1), \ldots, d)$$

$$\mathcal{H}_\tau = \sum_j \sum_{\mu \vdash j} d^{\binom{d(m+1)-j-1}{\ell(\mu)}} \binom{m+1}{j+1} \frac{(\ell(\mu)-1)!}{r_1!\, r_2! \cdots r_k!} h_\mu$$

$r_i :=$ # Parts of size i in μ

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Proposition

$$\tau = (dm, d(m-1), \ldots, d)$$

$$\mathcal{H}_\tau = 1 + d\binom{m+1}{2} h_1 + d\binom{m+1}{3} h_2$$

$$+ d\binom{d(m+1)-3}{1}\binom{m+1}{3} h_{11}$$

$$+ \quad \cdots \quad + d\, h_m$$

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Partages Triangulaires

En collaboration avec
Mikhaïl Mazin
ArXiv: 2203.15942

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BLASIAK, HAIMAN, MORSE PUN, SEELINGER

UNE SUITE D'ARTICLES

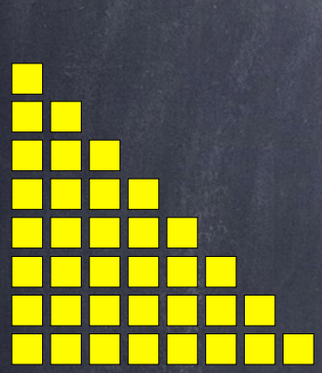
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2102.07931

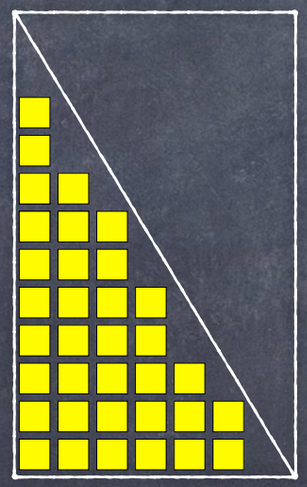
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ÉVOLUTION DE LA COMBINATOIRE DE CATALAN

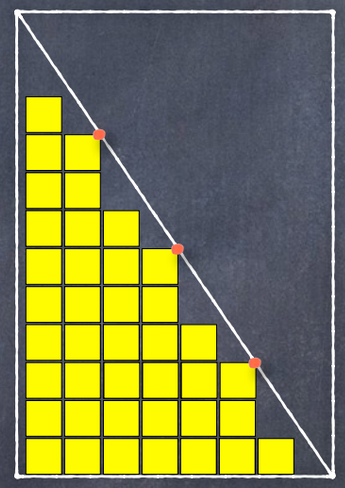
CLASSIQUE ⊆ RATIONNELLE ⊆ RECTANGULAIRE ⊆ TRIANGULAIRE ⊆ PARTITIONNELLE





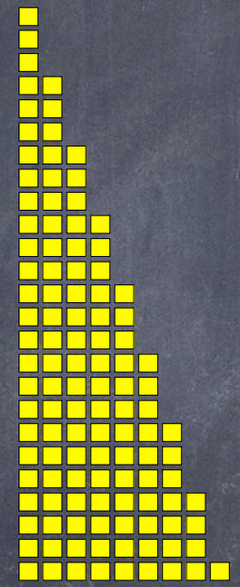
b

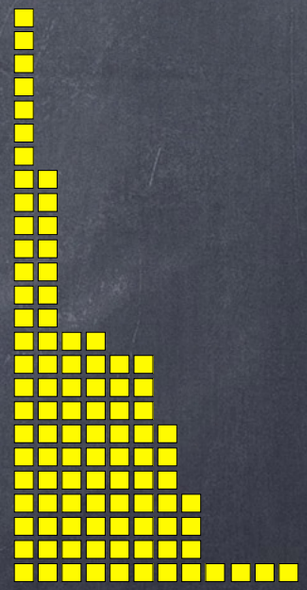
a



bk

ak



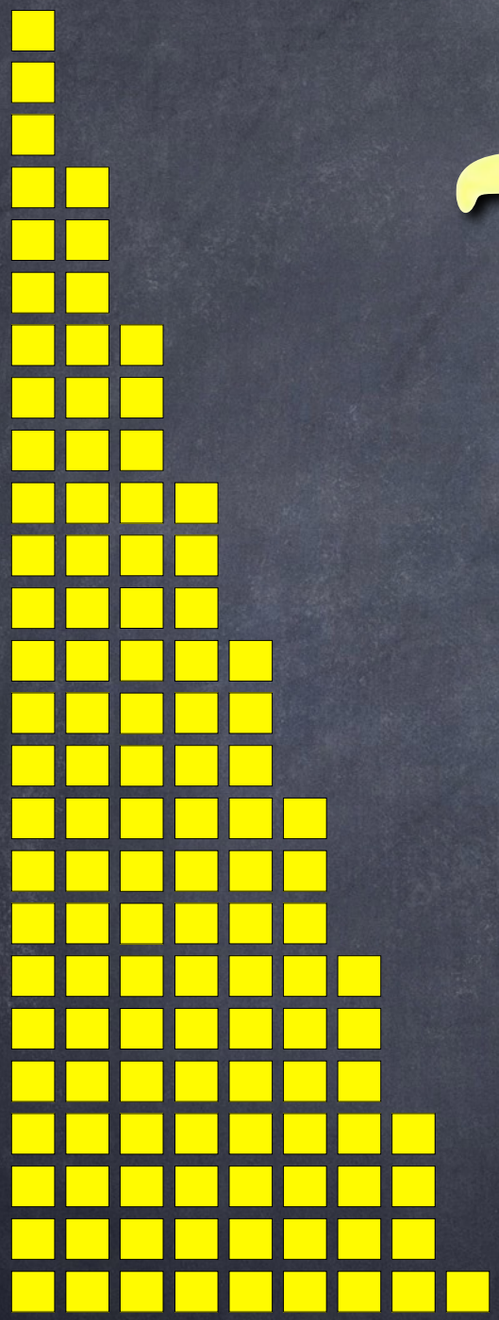


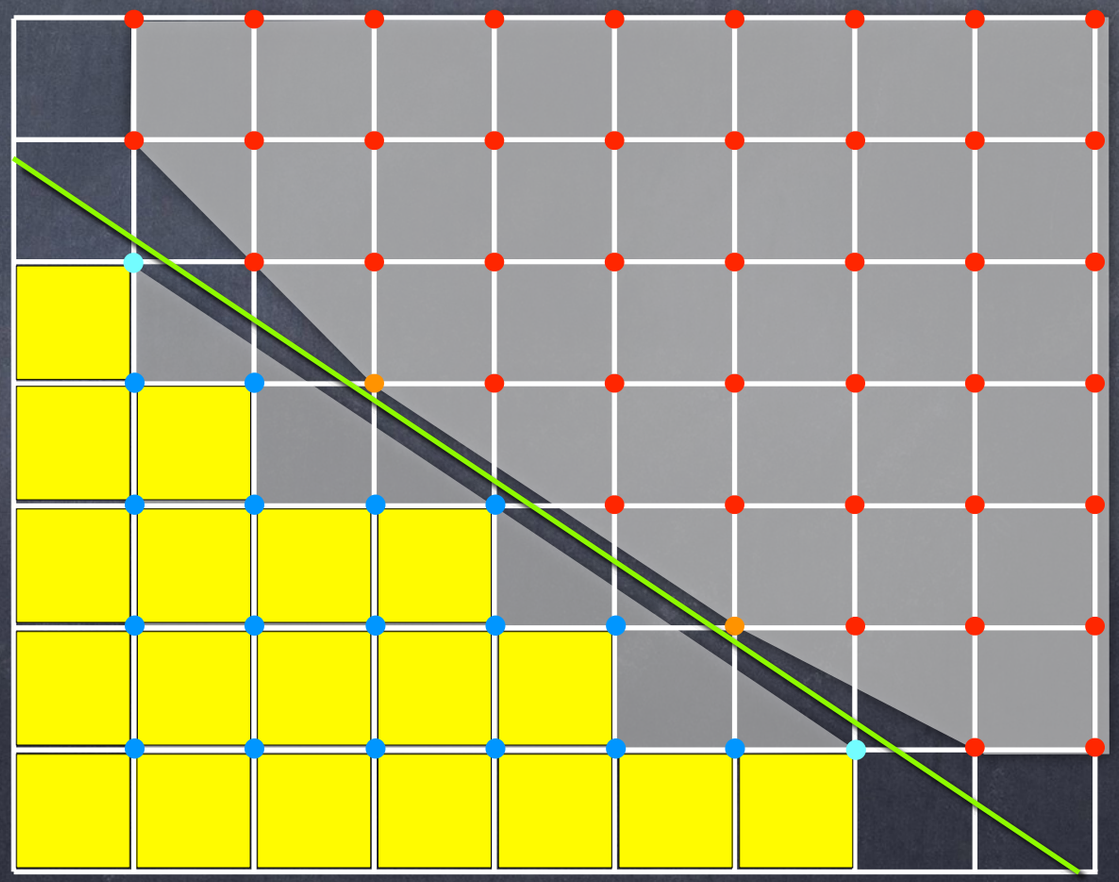
$a, b, k \in \mathbb{N} \quad (a,b) = 1$

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TREILLIS T-TAMARI

COMPLEXES ASSOCIAÈDRAUX

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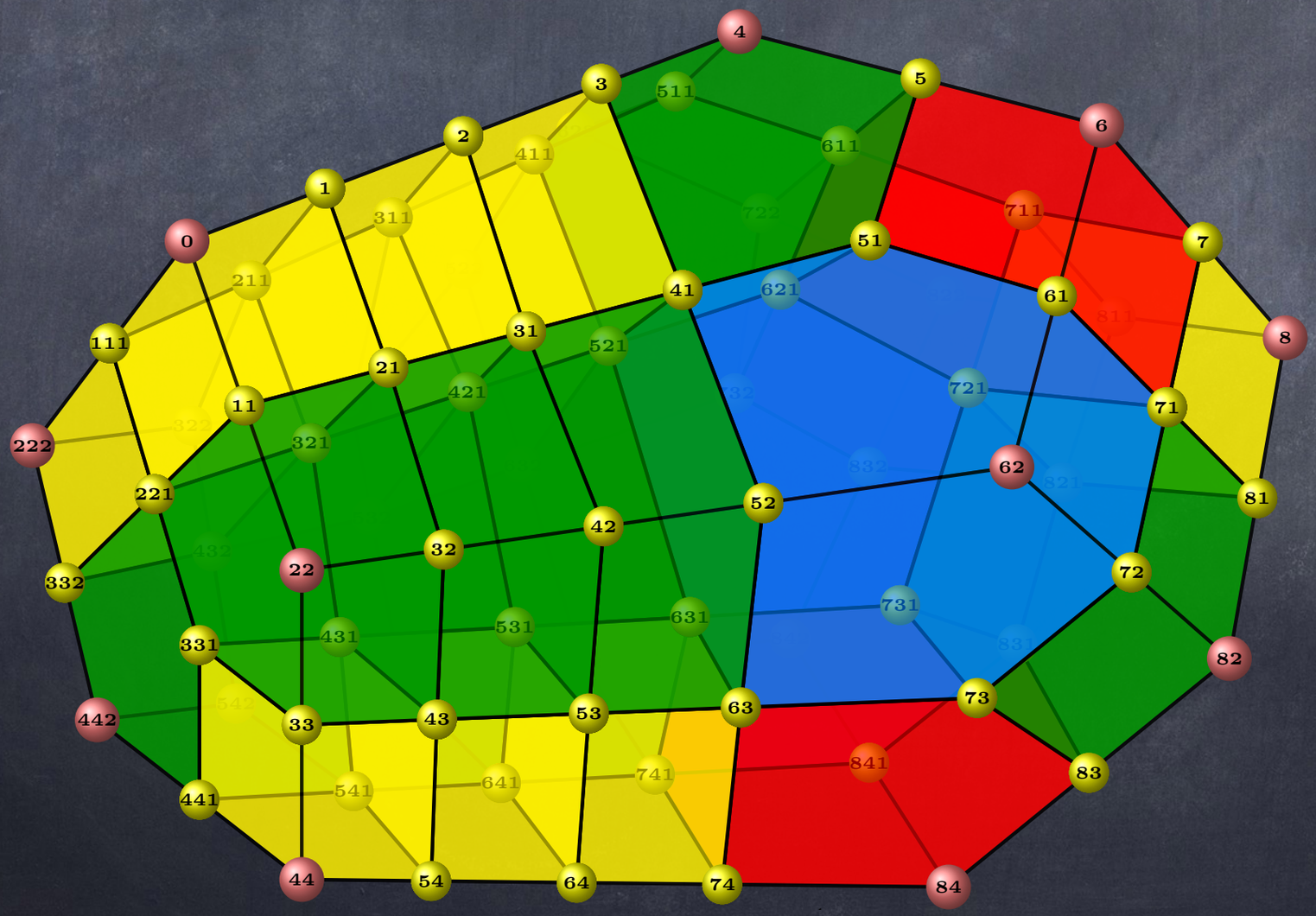
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FiN





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