Trailing the dovetail shuffle to its lair.


Rarely does a new mathematical result make both the New York Times and the front page of my local paper, and even more rarely is your reviewer asked to speak on commercial radio about a result, but such activity was caused by the preprint of this paper. In layman’s terms, it says you should shuffle a deck of cards seven times before playing. More technically, the usual way people shuffle is called a riffle shuffle, and a natural mathematical model of a random shuffle is to assume all possible riffle shuffles are equally likely. With this model one can ask how close is $k$ shuffles of an $n$-card deck to the uniform distribution on all $n!$ permutations, where “close” is measured by variation distance. It was previously known that, as $n \to \infty$, one needs $k(n) \sim \frac{3}{2} \log_2 n$ shuffles to get close to uniform. This paper gives an elegant and careful treatment based on an explicit formula for the exact distance $d(k, n)$ to uniformity. To quote the abstract: “Key ingredients are the analysis of a card trick and the determination of the idempotents of a natural commutative subalgebra in the symmetric group algebra.”

David J. Aldous