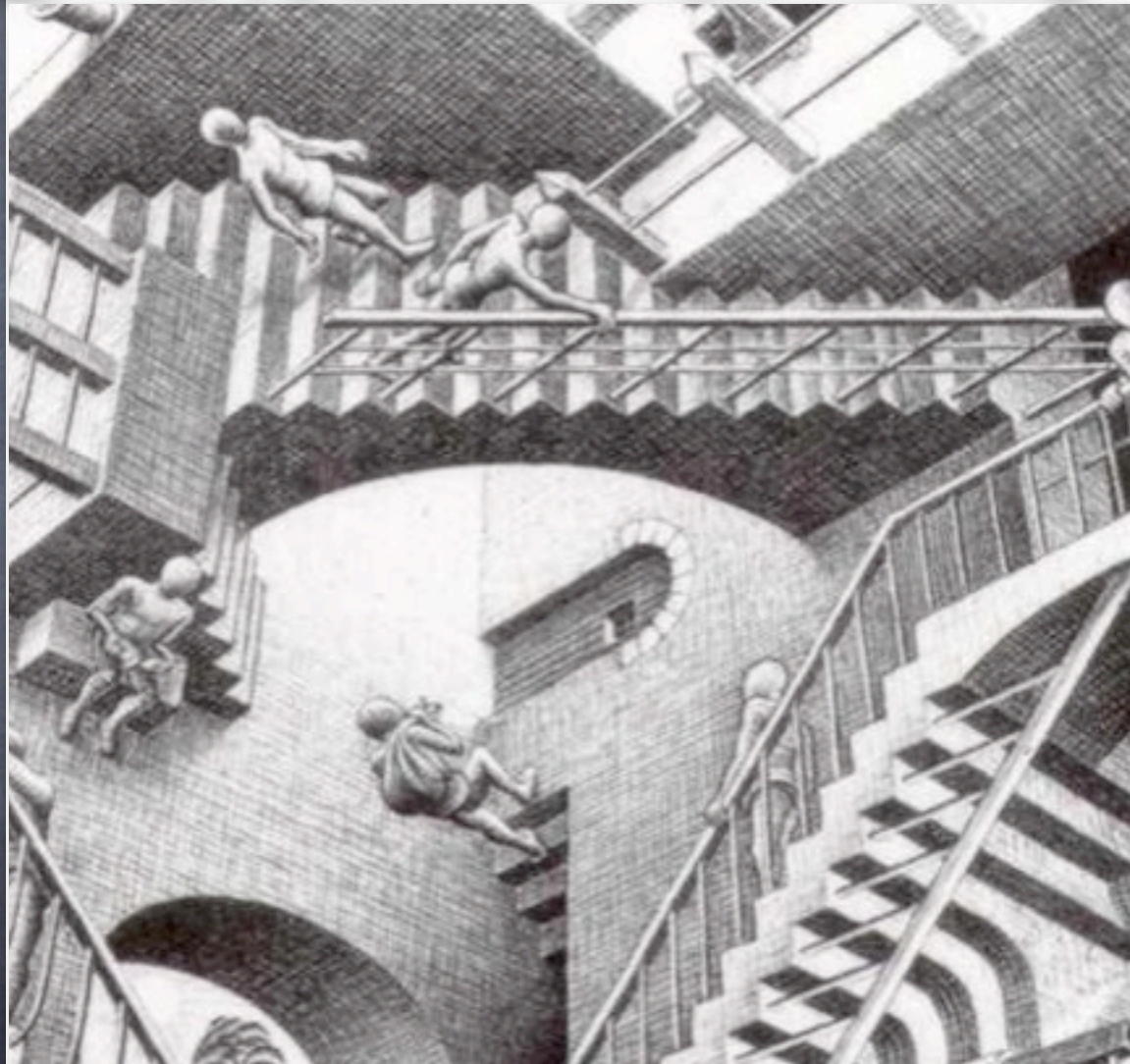


# LES MATHÉMATIQUES AU SECOURS DE L'ART D'ESCHER



Relativité,  
1953



# LES MATHÉMATIQUES AU SECOURS DE L'ART D'ESCHER



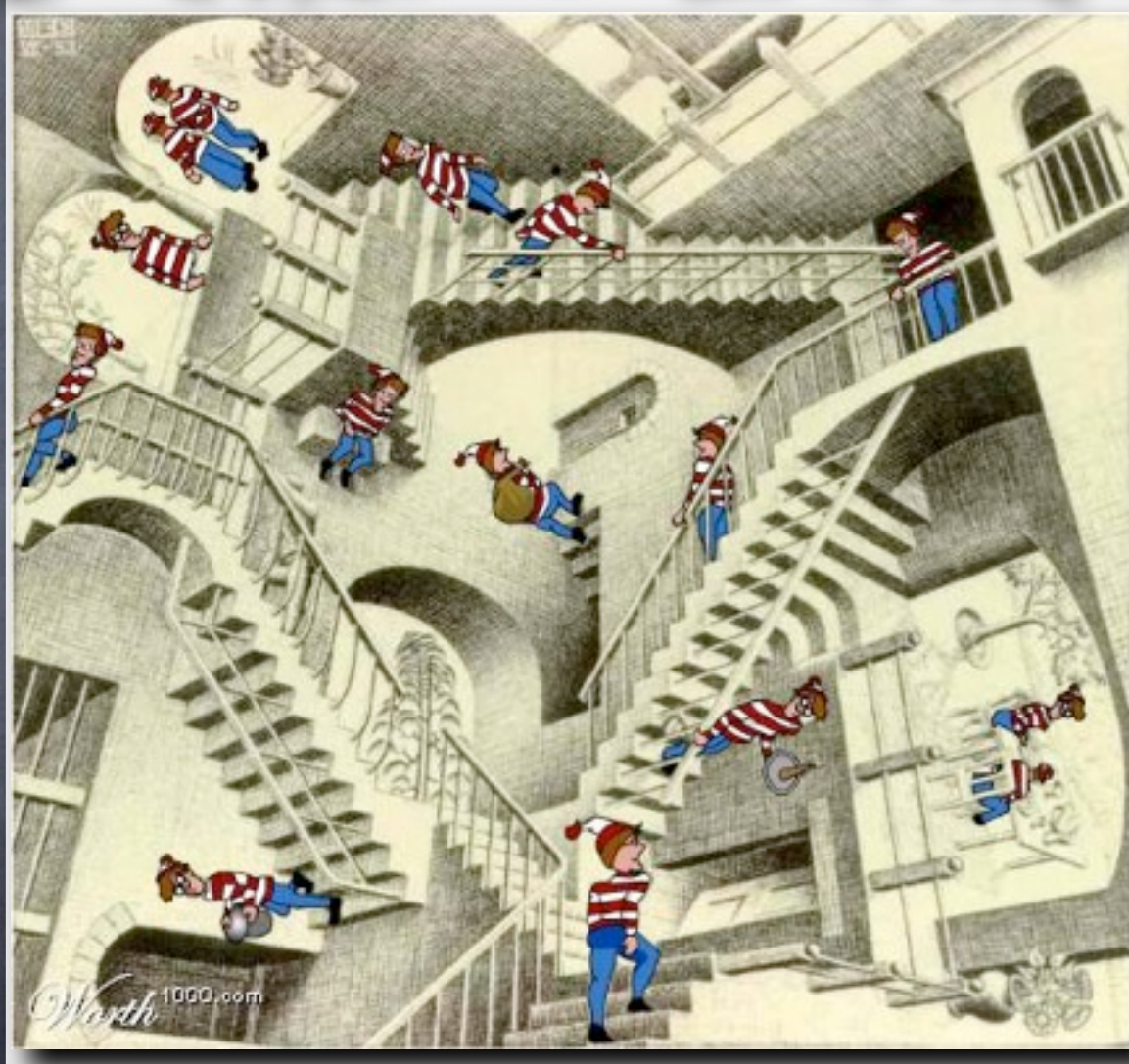
Andrew Lipson  
et  
Daniel Shiu,  
2000

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# LES MATHÉMATIQUES AU SECOURS DE L'ART D'ESCHER



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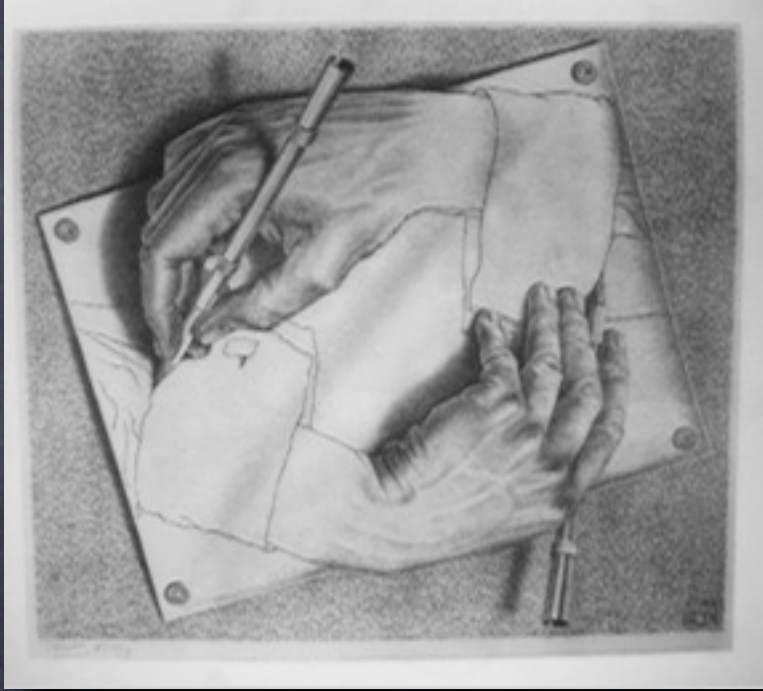
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Qui EST ESCHER



# MAURITS CORNELIS ESCHER (1898 - 1972)



Drawing Hands, 1948



Autoportrait, 1943



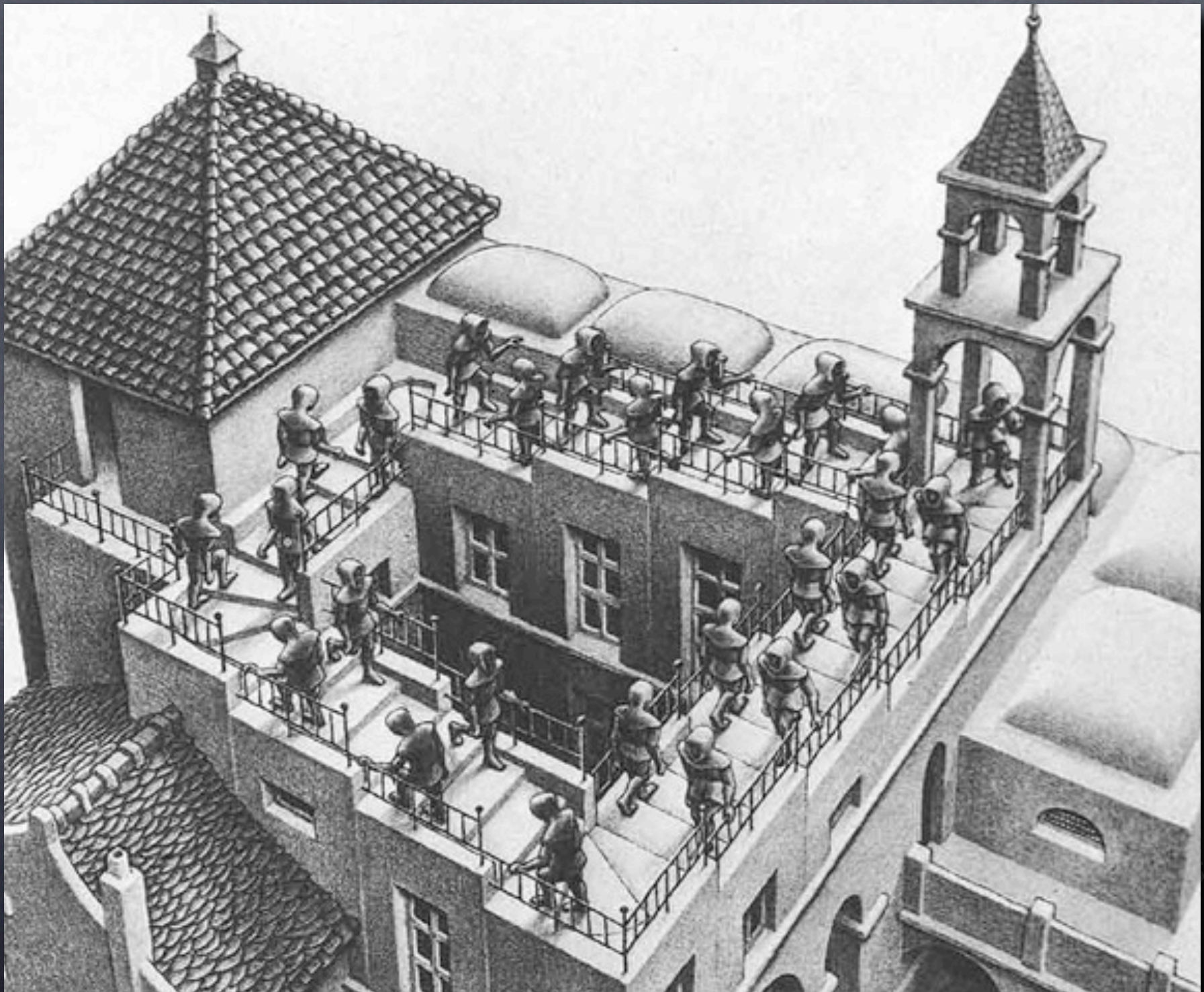


Maison de ferme à Ravello, Rome 1931.

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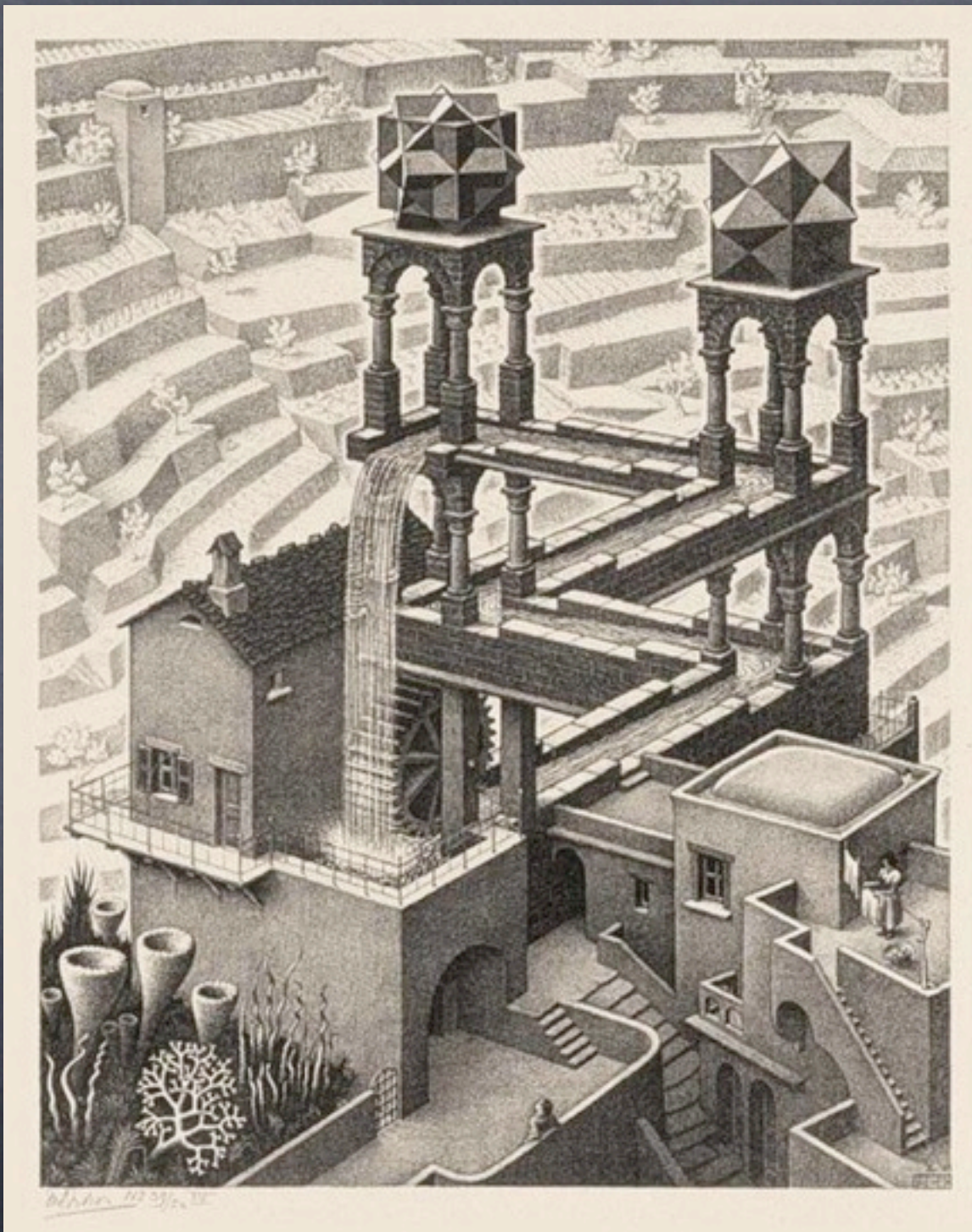


cente,

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Chute,  
1961

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# SOURCES D'INSPIRATION

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# L'ALHAMBRA (GRENADE, ESPAGNE.)

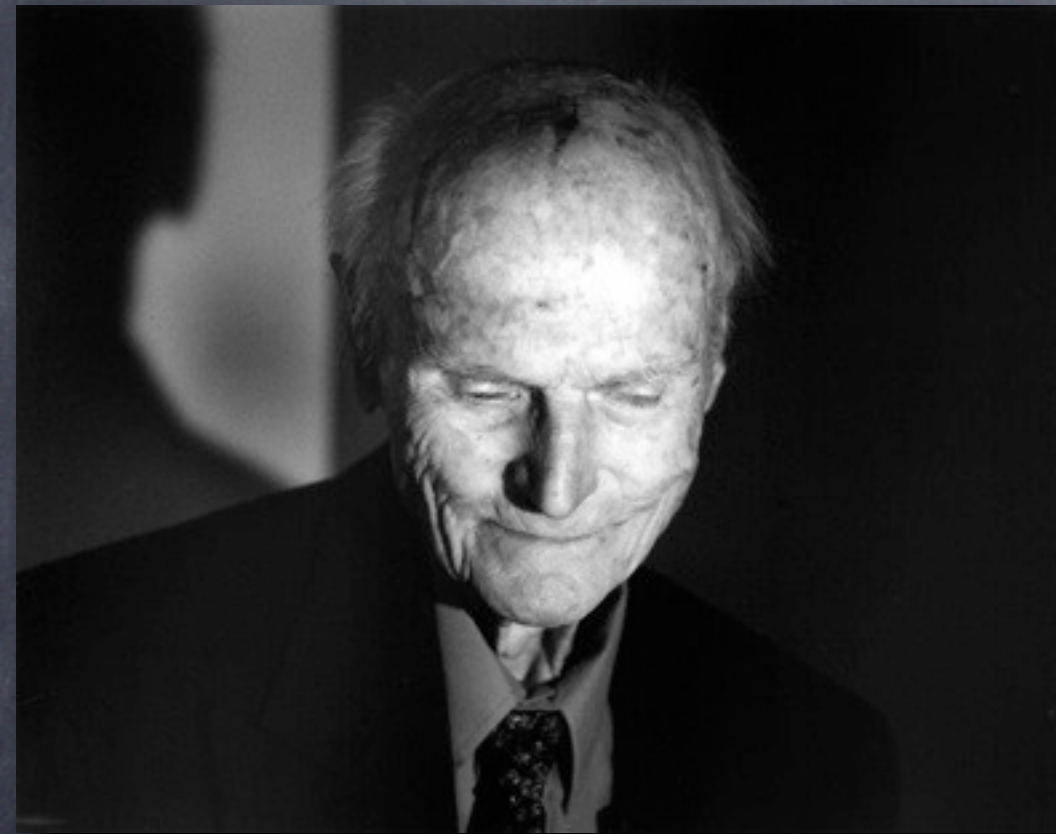


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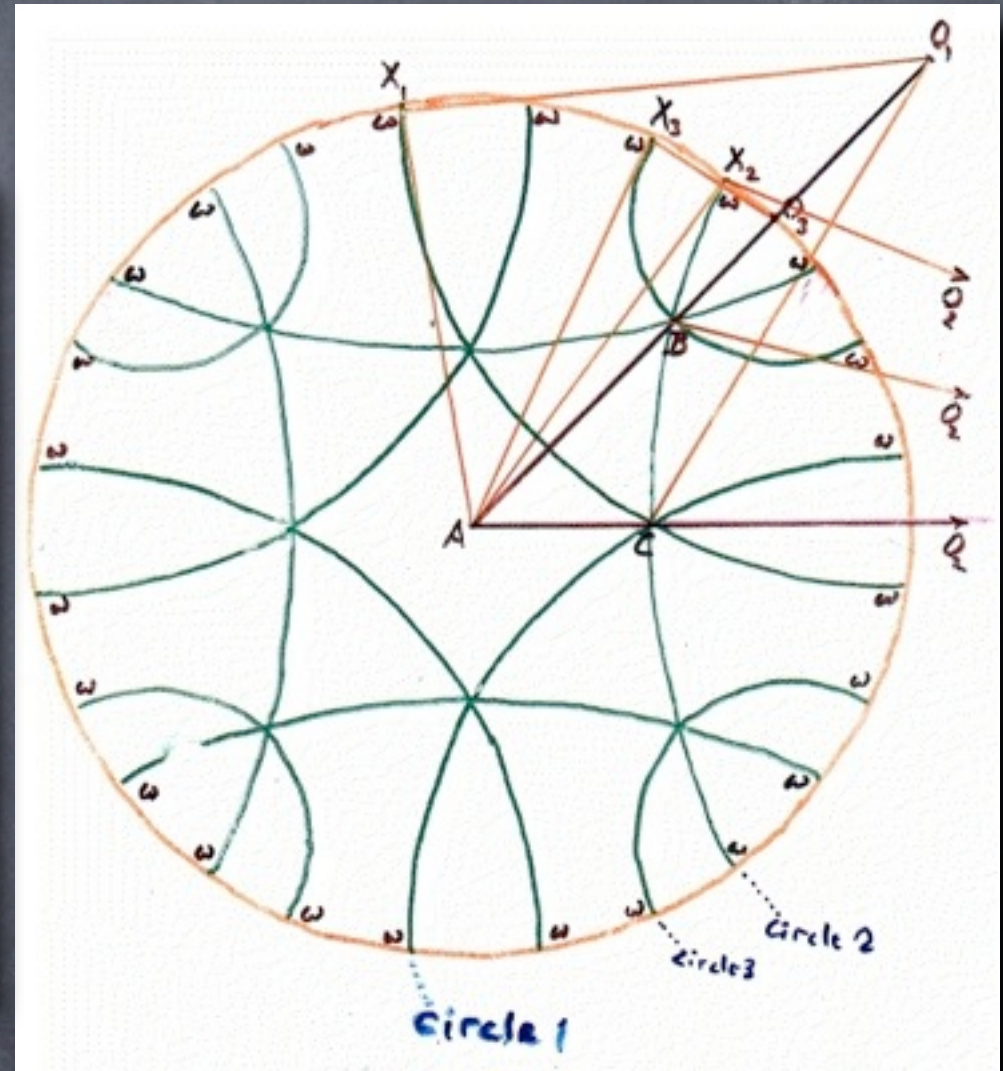
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# UNE AMITIÉ DATANT DE 1954



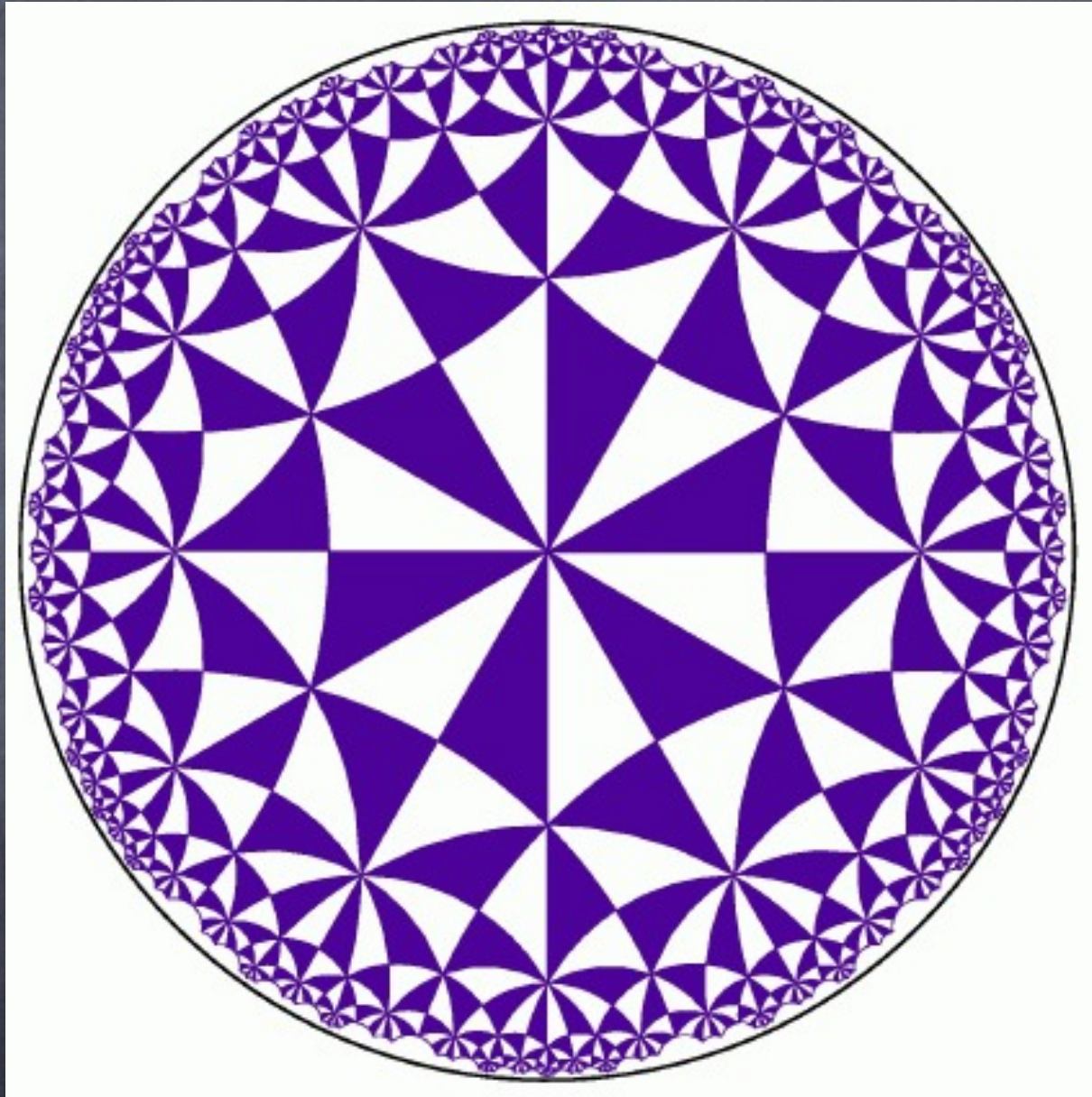
H.S.M. Coxeter (1907–2001)  
Univ. Toronto



Plan hyperbolique



# GÉOMÉTRIE HYPERBOLIQUE



**Crystal symmetry and its generalizations**

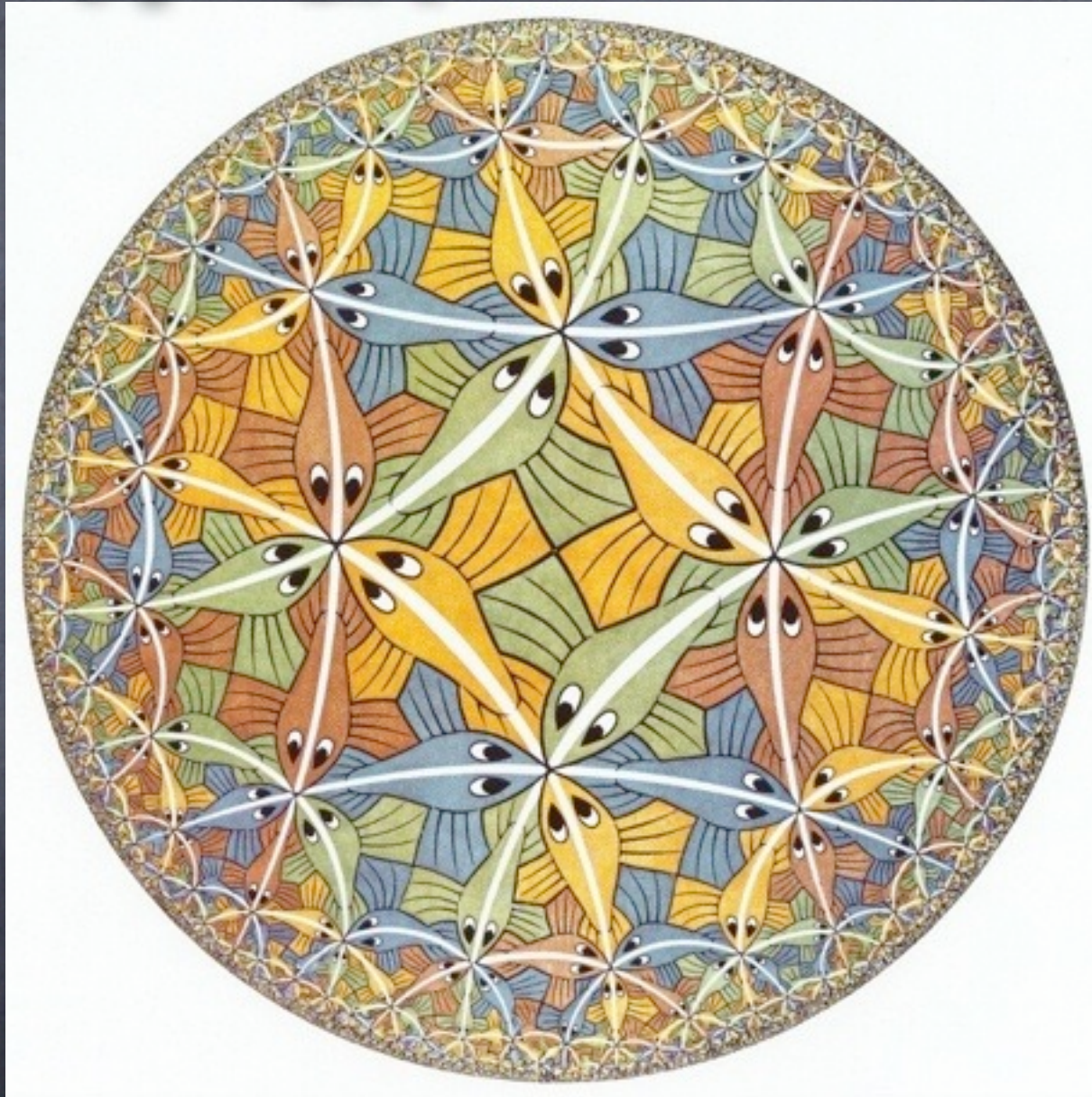
HSM Coxeter - Trans. Royal Soc. Canada (3), 1957, 1-

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# À LA ESCHER



Cercle limite  
III, 1959

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Ruban de Moebius





Ruban de Moebius II,  
1963

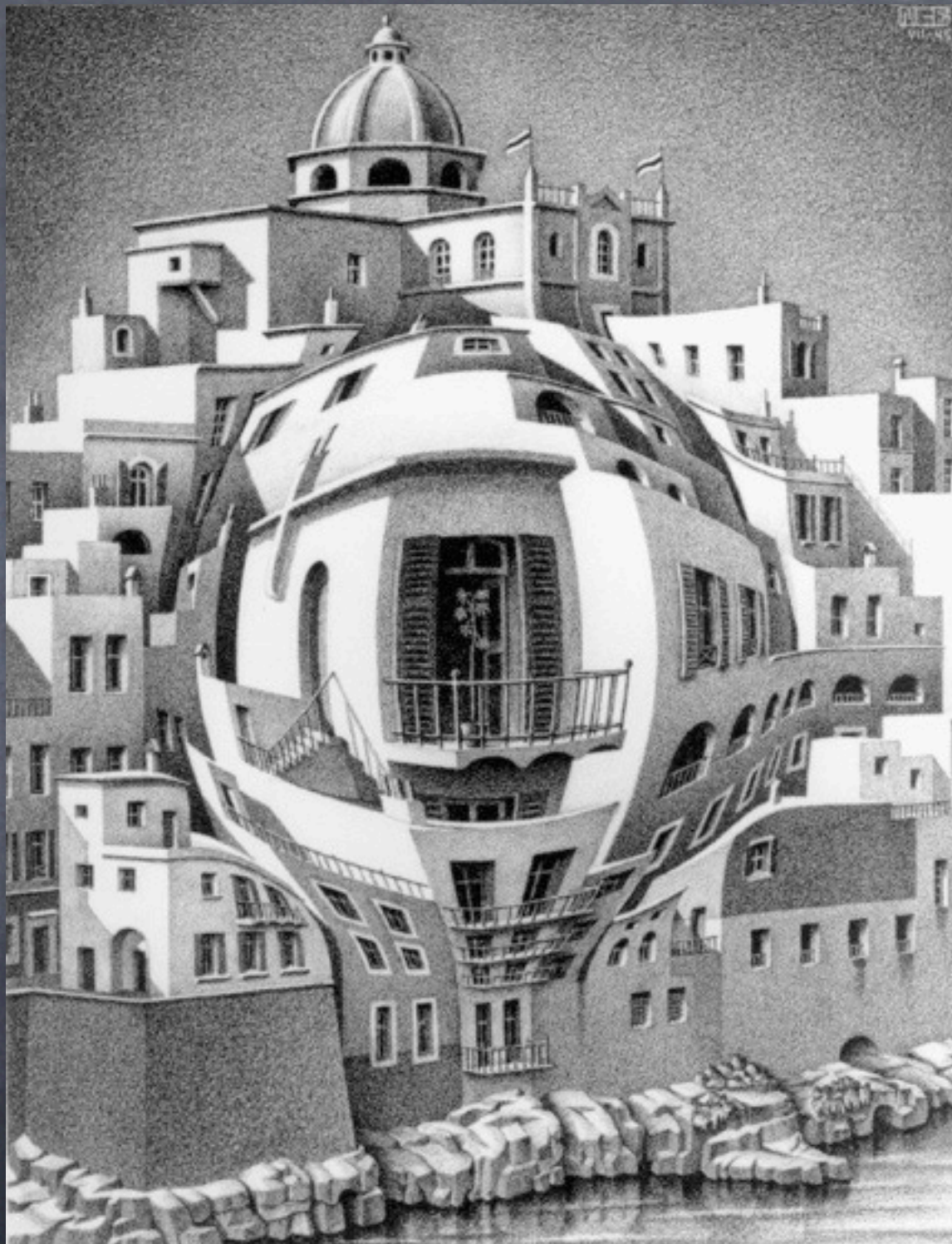


# L'ART DE LA DÉFORMATION

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Balcon, 1945

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# UNE VIEILLE TRADITION



Les ambassadeurs,  
Holbein (1533)

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# NOTRE HISTOIRE D'AUJOURD'HUI

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# "PRETENTENTONSTELLING" (M.C. ESCHER, 1956)



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# HENDRIK W. LENSTRA

Mathematical Institute  
Universeit Leiden  
Netherlands

Department of Mathematics  
University of California at Berkeley  
USA

Topics: Algebra, number theory, algorithms



Bart de Smit

## Titles of major publications

- Factoring integers with **elliptic curves** Annals of Mathematics 126 (1987), 649-673.
- Algorithms in algebraic number theory, Bulletin of the American Mathematical Society 26 (1992), 211-244.
- The development of the number field sieve (with A.K. Lenstra), Lecture Notes in Mathematics 1554, Springer-Verlag, Heidelberg, 1993.
- Artin reciprocity and Mersenne primes (with P. Stevenhagen), Nieuw Archief voor Wiskunde (5) 1 (2000), 44-54.
- Flags and lattice basis reduction, European congress of mathematics, Birkhäuser, Basel, 2001.

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# UNE REVUE DE PRESSE

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## Mathematician Fills in a Blank for a Fresh Insight on Art

By SARA ROBINSON

**O**n a flight to the Netherlands, Dr. Hendrik Lenstra, a mathematician, was leafing through an airline magazine when a picture of a lithograph by the Dutch artist M. C. Escher caught his eye.

Titled "Print Gallery," it provides a glimpse through a row of arching windows into an art gallery, where a man is gazing at a picture on the wall. The picture depicts a row of Mediterranean-style buildings with turrets and balconies, fronting a quay on the island of Malta.

As the viewer's eye follows the line of buildings to the right, it begins to bulge outward and twist downward, until it sweeps around to include the art gallery itself. In the center of the dizzying whorl of buildings, ships and sky, is a large, circular patch that Escher left blank. His signature is scrawled across it.

As Dr. Lenstra studied the print he found his attention returning again and again to that central patch, puzzling over the reason Escher had not filled it in. "I wondered whether if you continue the lines inward, if there's a mathematical problem that cannot be solved," he said. "More generally, I also wondered what the structure is behind the picture: how would I, as a mathematician, make a picture like that?"



# Artful Mathematics: The Heritage of M. C. Escher

## Celebrating Mathematics Awareness Month

In recognition of the 2003 Mathematics Awareness Month theme "Mathematics and Art", this article brings together three different pieces about intersections between mathematics and the artwork of M. C. Escher. For more information about Mathematics Awareness Month, visit the website <http://mathforum.org/mam/03/>. The site contains materials for organizing local celebrations of Mathematics Awareness Month.

### The Mathematical Structure of Escher's Print Gallery

B. de Smit and H. W. Lenstra Jr.

In 1956 the Dutch graphic artist Maurits Cornelis Escher (1898–1972) made an unusual lithograph with the title *Prententoonstelling*. It shows a young man standing in an exhibition gallery, viewing a print of a Mediterranean seaport. As his eyes follow the quayside buildings shown on the print from left to right and then down, he discovers among them the very same gallery in which he is standing. A circular white patch in the middle of the lithograph contains Escher's monogram and signature.

What is the mathematics behind *Prententoonstelling*? Is there a more satisfactory way of filling in the central white hole? We shall see that the lithograph can be viewed as drawn on a certain elliptic curve over the field of complex numbers and

B. de Smit and H. W. Lenstra Jr. are at the Mathematisch Instituut, Universiteit Leiden, the Netherlands. H. W. Lenstra also holds a position at the University of California, Berkeley. Their email addresses are [desmit@math.leidenuniv.nl](mailto:desmit@math.leidenuniv.nl) and [hw1@math.leidenuniv.nl](mailto:hw1@math.leidenuniv.nl).



M. C. Escher's "Prententoonstelling" © 2003 Cordon Art B. V.-Baam-Holland. All rights reserved.

Figure 1. Escher's lithograph "Prententoonstelling" (1956).

deduce that an idealized version of the picture repeats itself in the middle. More precisely, it contains a copy of itself, rotated clockwise by  $157.6255960832 \dots$  degrees and scaled down by a factor of  $22.5836845286 \dots$ .

#### Escher's Method

The best explanation of how *Prententoonstelling* was made is found in *The Magic Mirror of M. C. Escher* by Bruno Ernst [1], from which the following quotations and all illustrations in this section are taken. Escher started "from the idea that it must...be possible to make an annular bulge," "a cyclic expansion...without beginning or end." The realization of this idea caused him "some almighty headaches." At first, he "tried to put his idea into

# Notices

of the American Mathematical Society

Avril 2003



# LA DÉMARCHE D'ESCHER

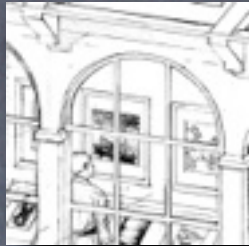
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# CE QU'ON SAIT DE LA DÉMARCHE D'ESCHER

1)



Une étude autosimilaire

2)



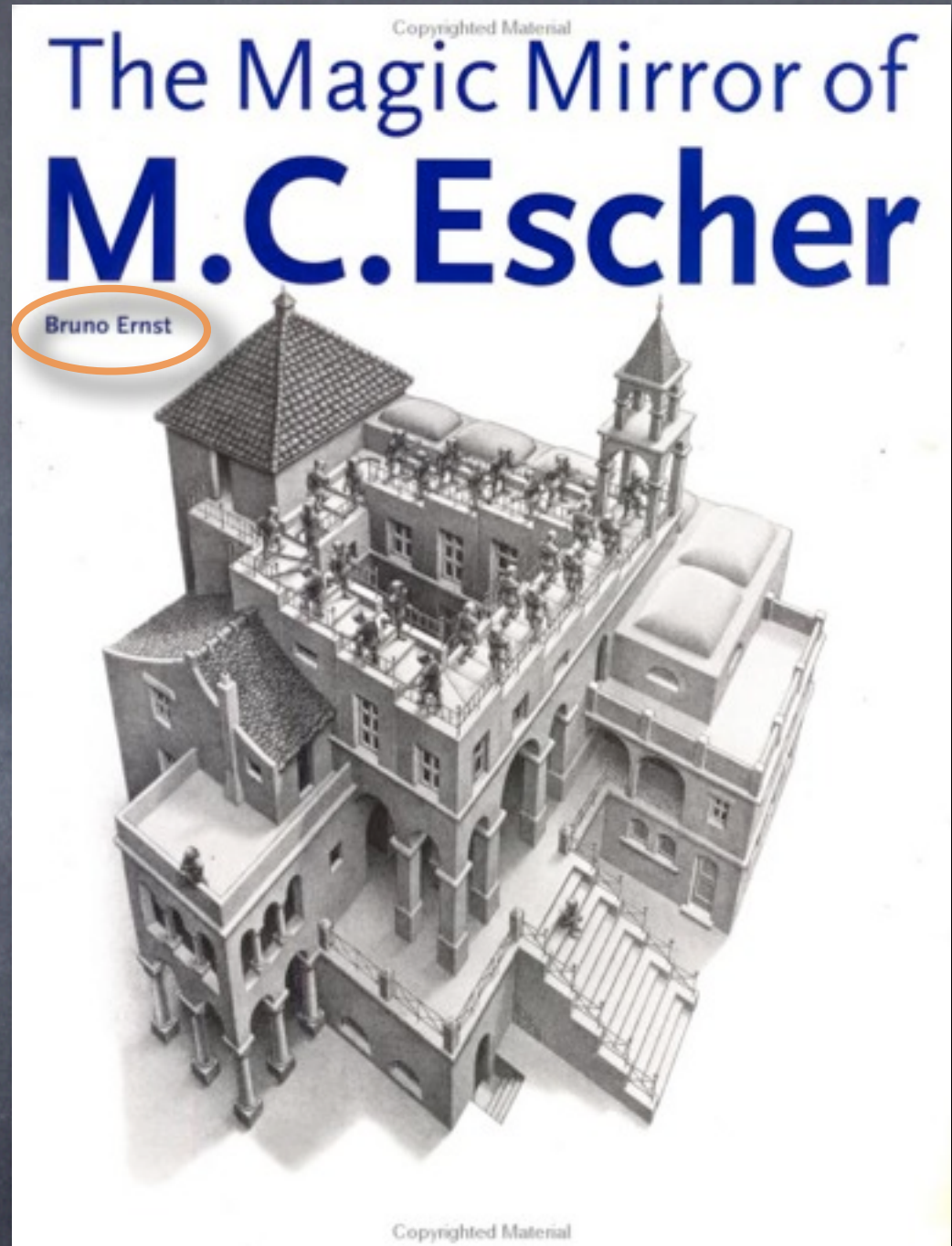
Une grille de "torsion"

3)

Son intention explicite de conserver  
l'apparence de "petits" carrés.



HANS DE Rijk



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# UNE ÉTUDE AUTO-SIMILAIRE



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# L'EFFET DROSTE





# L'EFFET "VACHE QUI RIT"





# L'EFFET "UMMAGUMMA"



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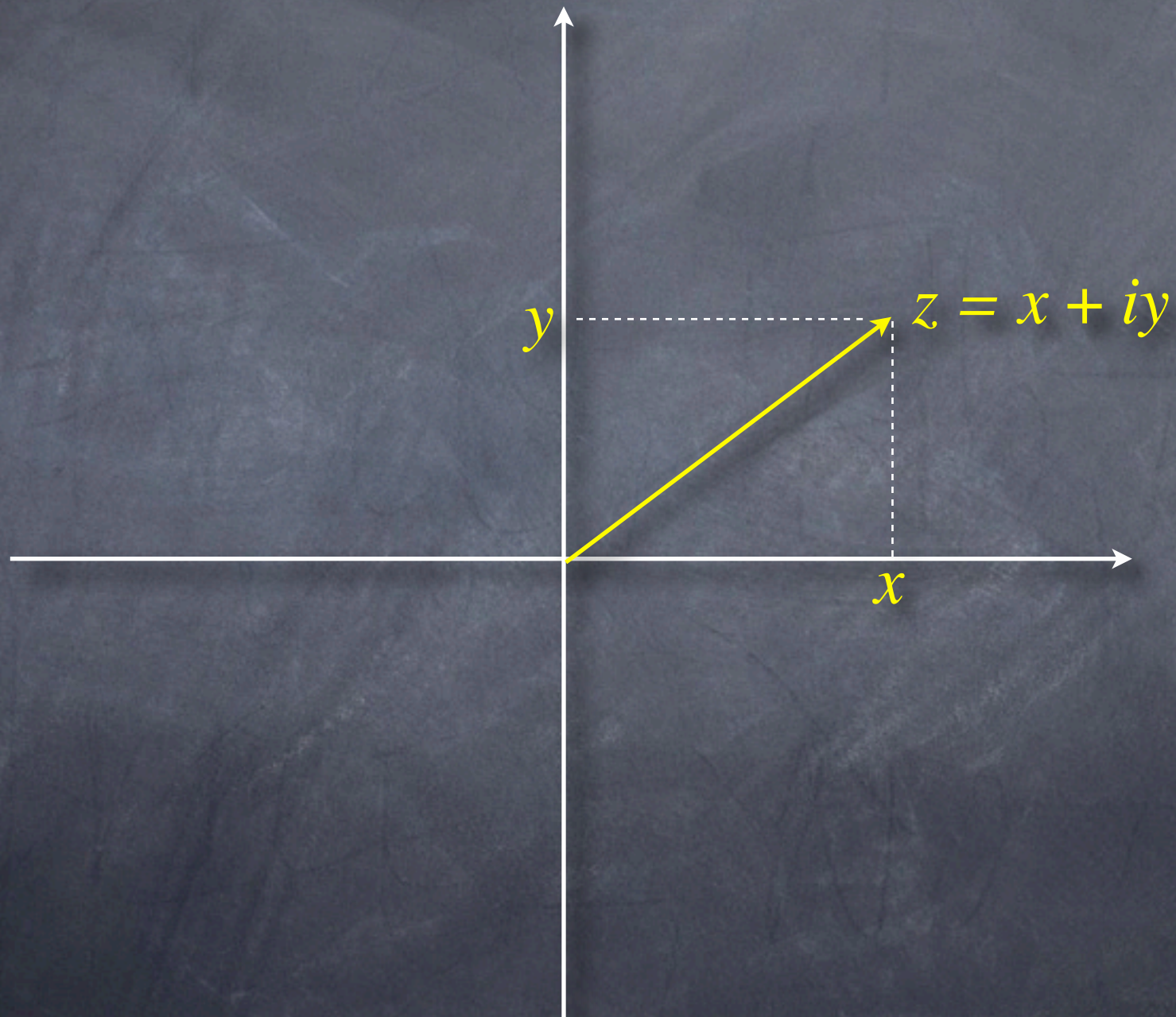
# MATHEMATISATION

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# NOMBRES COMPLEXES





# NOMBRES COMPLEXES $\mathbb{C}$

$$z = x + iy \quad i = \sqrt{-1}$$

$$(x + iy)(u + iv) = (xu - yv) + i(xv + yu)$$

$$\exp(x + iy) = e^x \cos(y) + i e^x \sin(y)$$

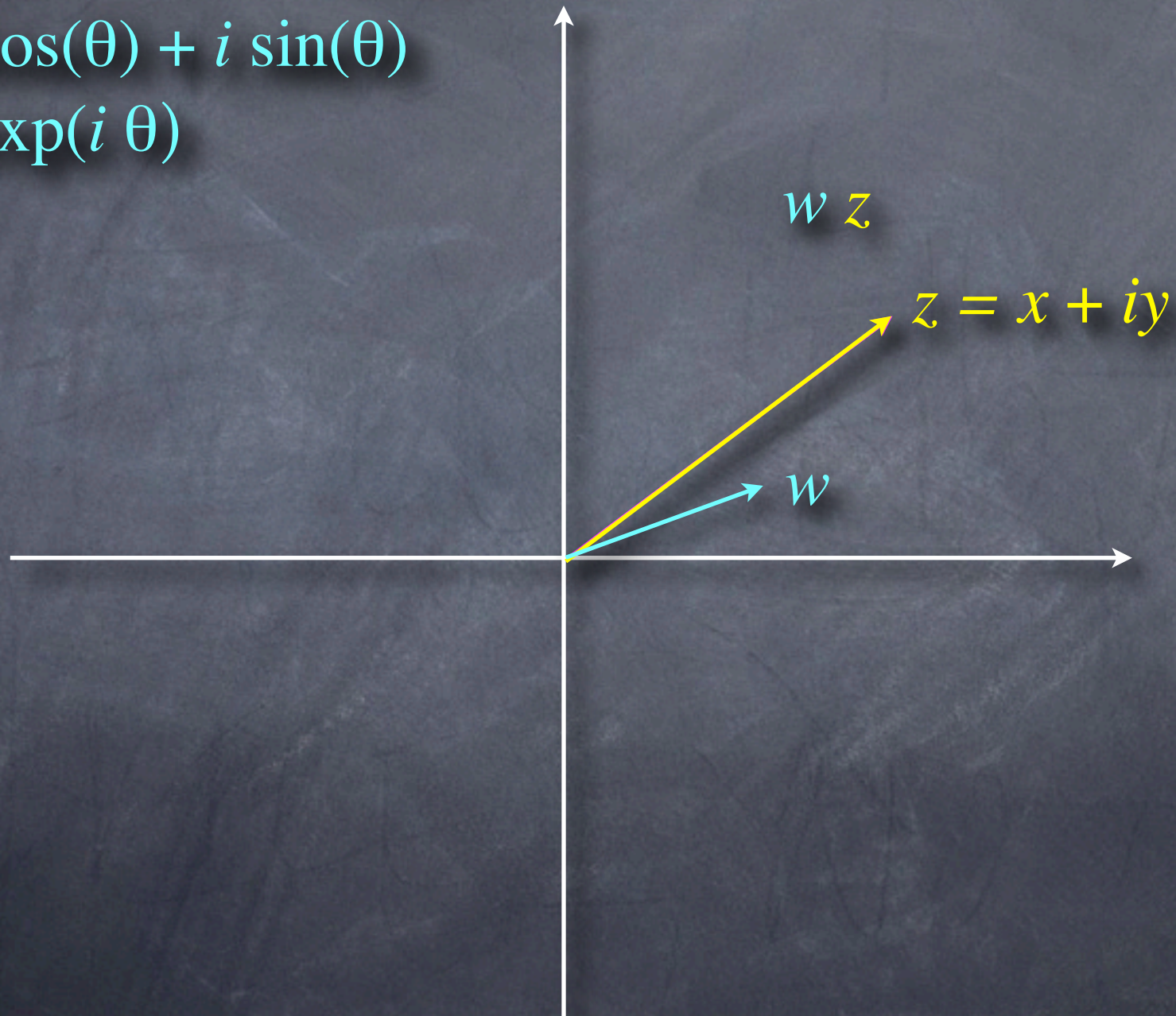
$$\frac{d}{dx} \exp(x + iy) = e^x \cos(y) + i e^x \sin(y)$$

$$\frac{d}{dy} \exp(x + iy) = -e^x \sin(y) + i e^x \cos(y)$$



# NOMBRES COMPLEXES

$$w = \cos(\theta) + i \sin(\theta) \\ = \exp(i \theta)$$





# UNE IMAGE

$$f : \mathbb{C} \rightarrow \{ \text{Noir} , \text{Blanc} \}$$

$$f(z) = f(256z)$$



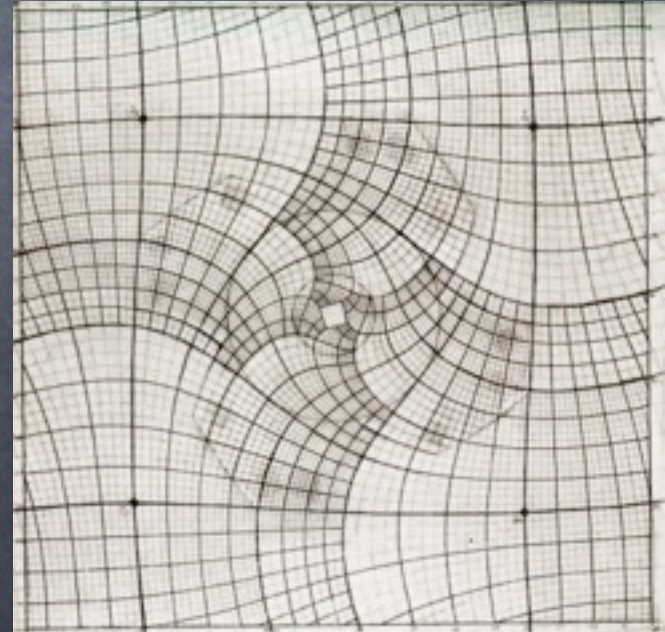
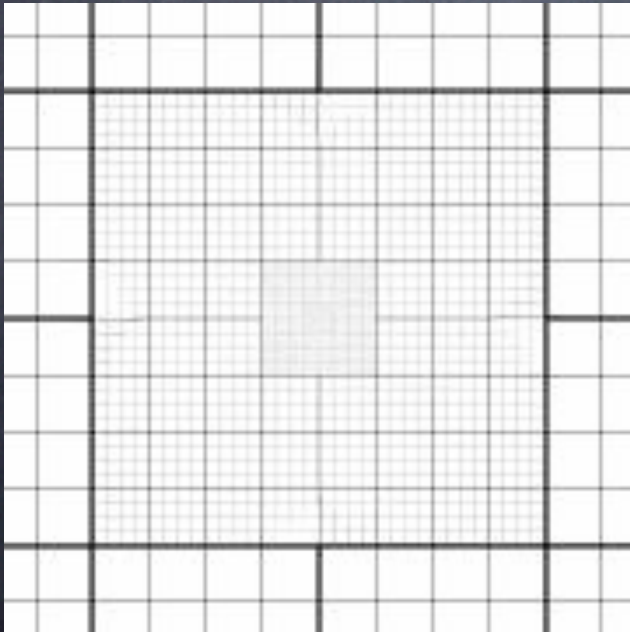
# LA GRILLE D'ESCHER

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# LA "TORSION" D'ESCHER



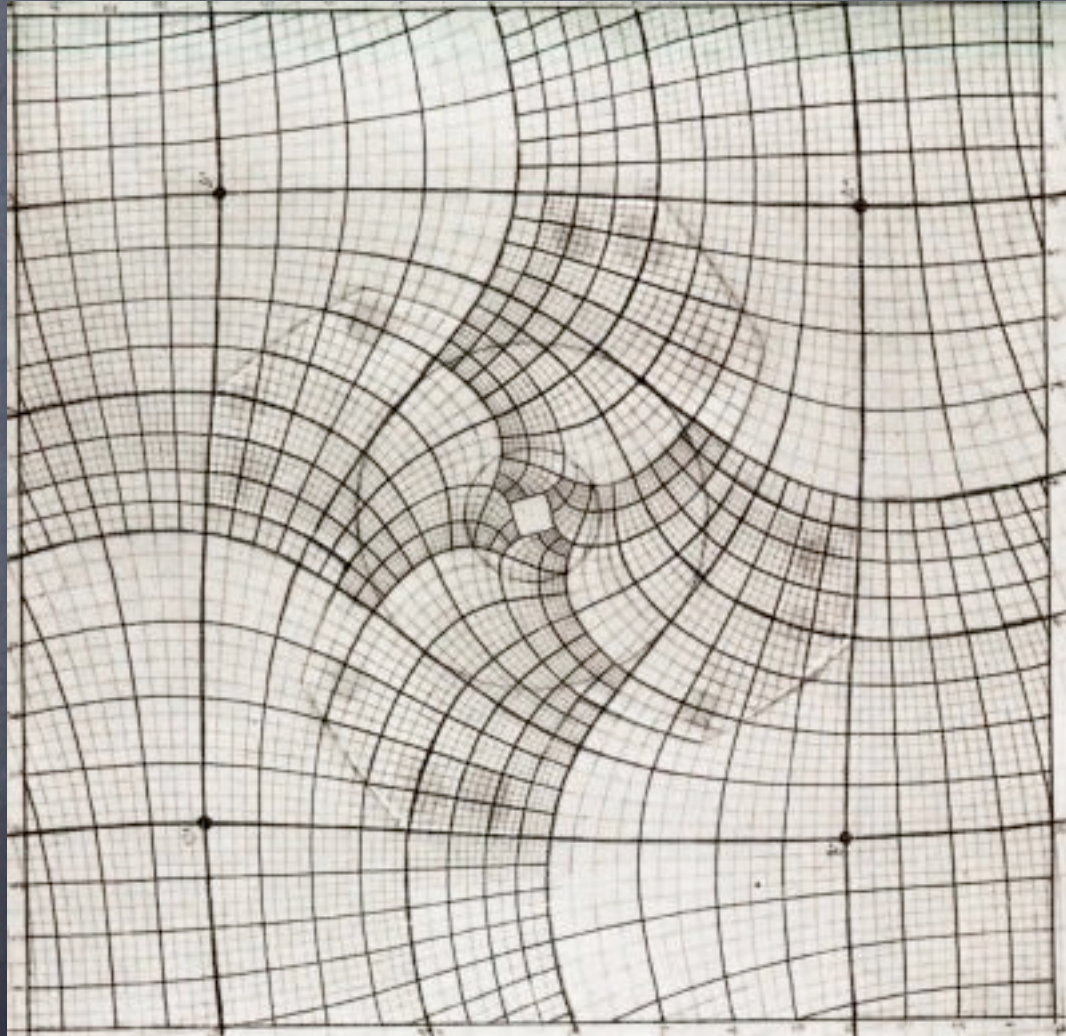
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# LA "TORSION" D'ESCHER

(CONSERVER L'APPARENCE DES CARRÉS)





# UNE AUTRE VIEILLE TRADITION



Albrecht Dürer, "Undersweysung der Messung", 1527.



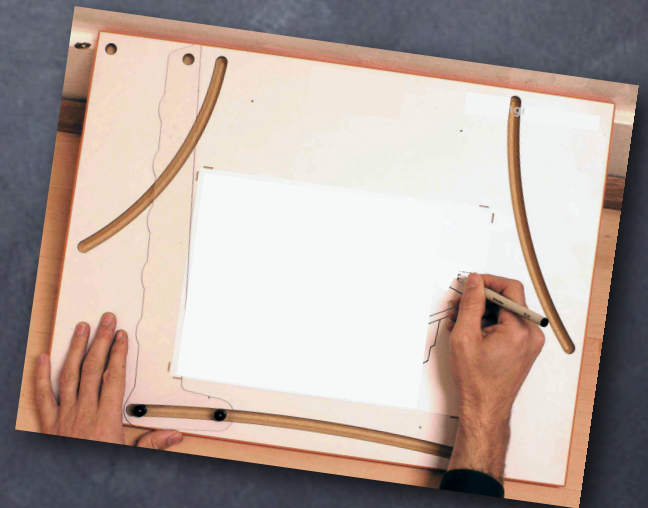
# UNE AUTRE VIEILLE TRADITION



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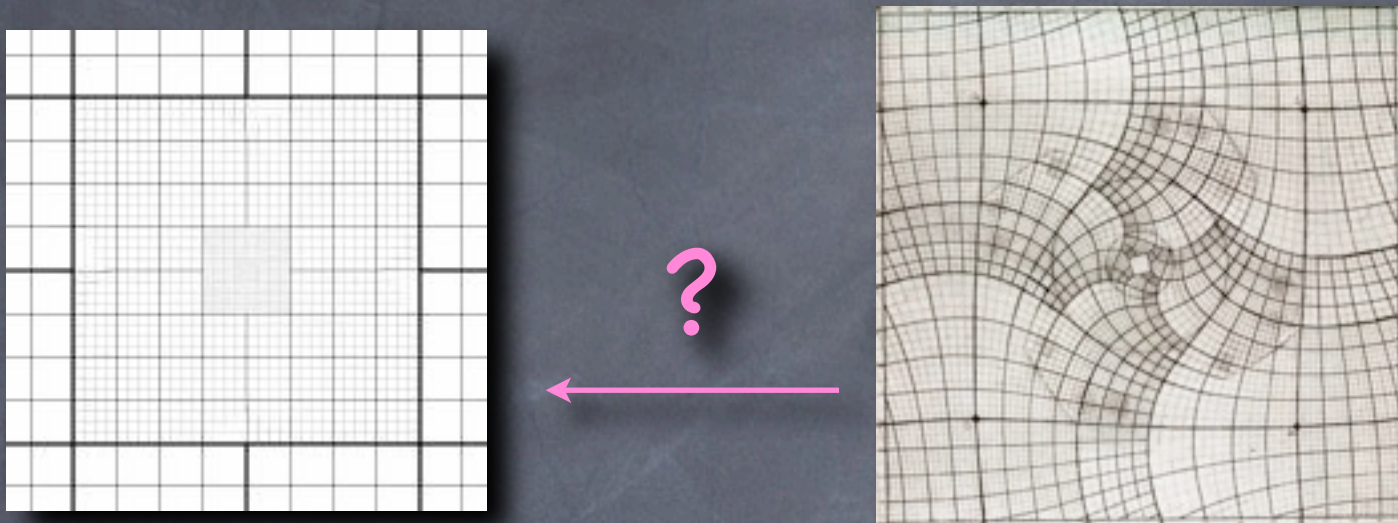


# LES MATHÉMATIQUES DE LA TORSION

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$f(z)$

$g(z)$

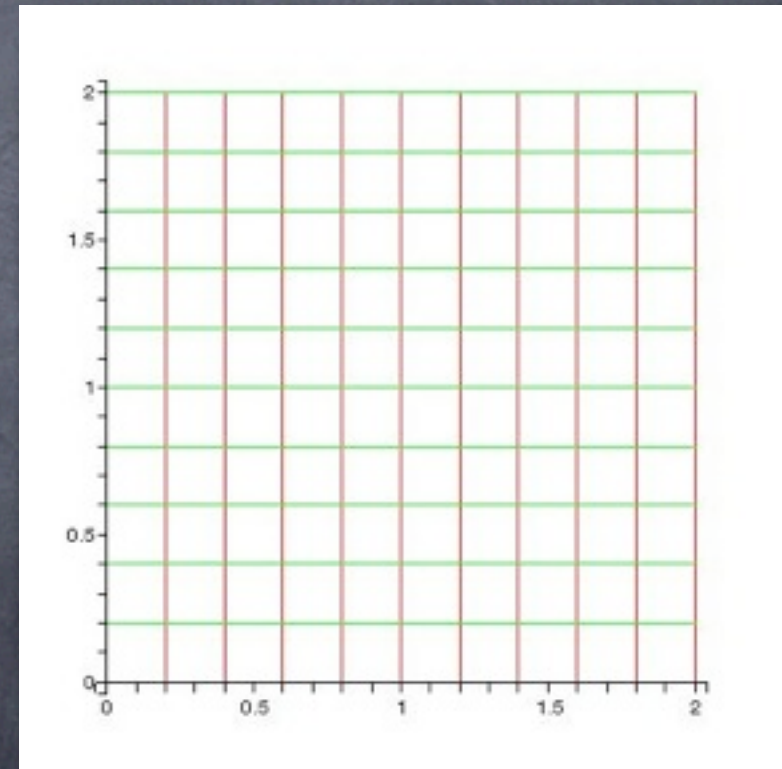
**{ Noir , Blanc }**



# TRANSFORMATIONS CONFORMES (RESPECTANT LES ANGLES)

$$h(x + iy) = u(x, y) + iv(x, y)$$

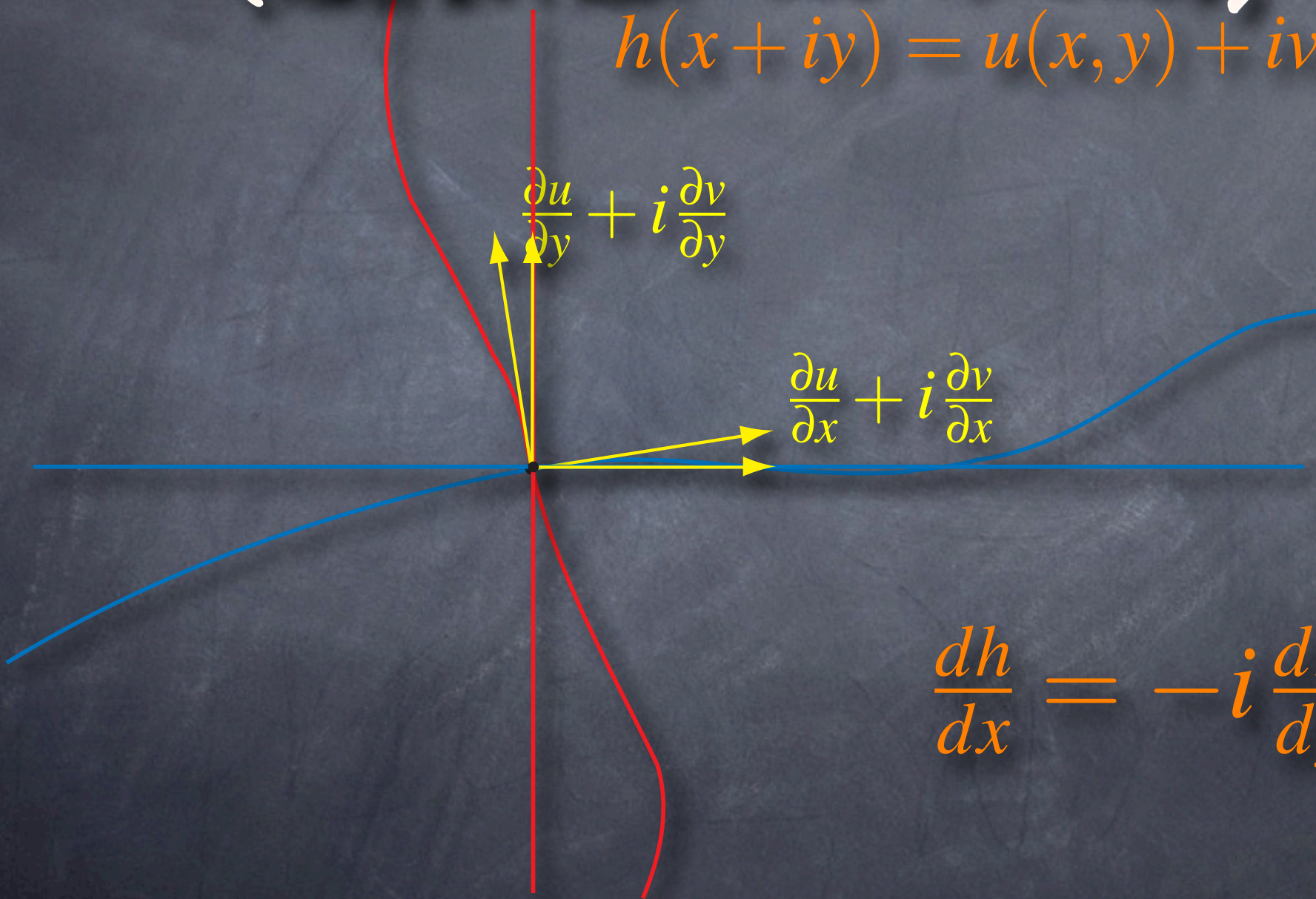
$$\sqrt{z} \longleftrightarrow z$$





# TRANSFORMATIONS CONFORMES (RESPECTANT LES ANGLES)

$$h(x + iy) = u(x, y) + iv(x, y)$$



$$\frac{dh}{dx} = -i \frac{dh}{dy}$$



# TRANSFORMATIONS CONFORMES (RESPECTANT LES ANGLES)

$$h(x + iy) = u(x, y) + iv(x, y)$$

$$\frac{dh}{dx} = -i \frac{dh}{dy}$$

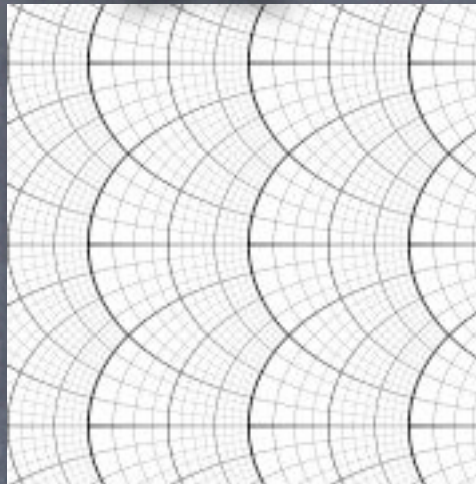
$$\sqrt{z} \longleftrightarrow z$$





# LOGARITHME / EXPONENTIELLE

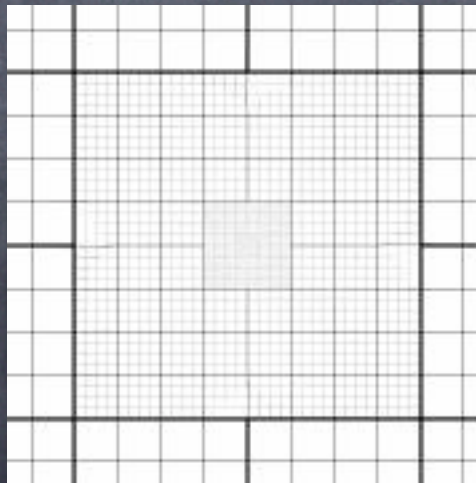
$$\log(256)$$



$$\left. \vphantom{\int} \right\} 2\pi i$$

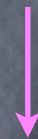


$$\exp(z)$$





# LOGARITHME / EXPONENTIELLE



$\exp(z)$





# LOGARITHME / EXPONENTIELLE EN ACTION



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# THÉORÈME :

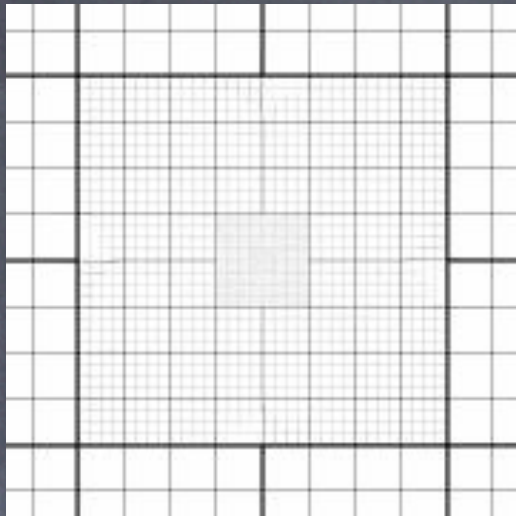
LES TRANSFORMATIONS CONFORMES  
DU TORE VERS LE TORE  
SONT DE LA FORME ...  
ET SEULEMENT DE CETTE FORME



Il n'y a qu'une et une seule  
transformation conforme qui  
satisfasse les contraintes  
imposées par Escher !

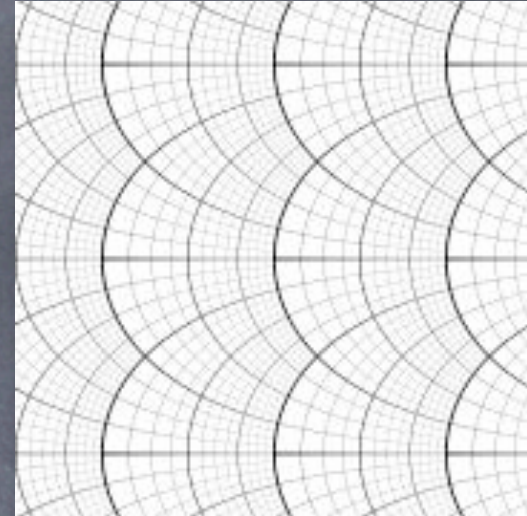


# "L'UNIQUE BONNE" TORSION



$\alpha z$

←



$\exp(z)$

↓

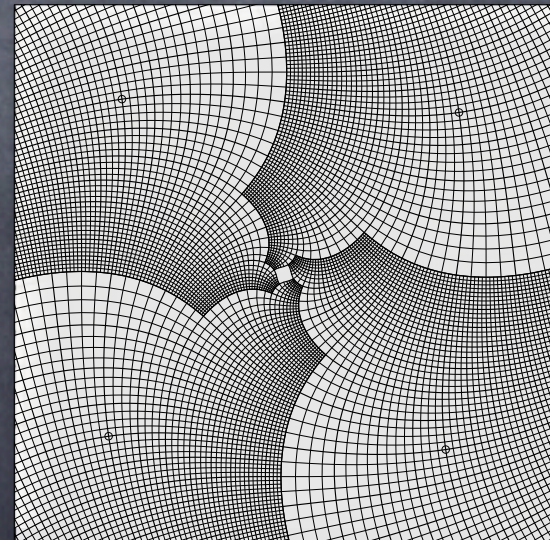
$$\alpha = \frac{2\pi i}{\log(256) + 2\pi i}$$

↓

$\exp(z)$

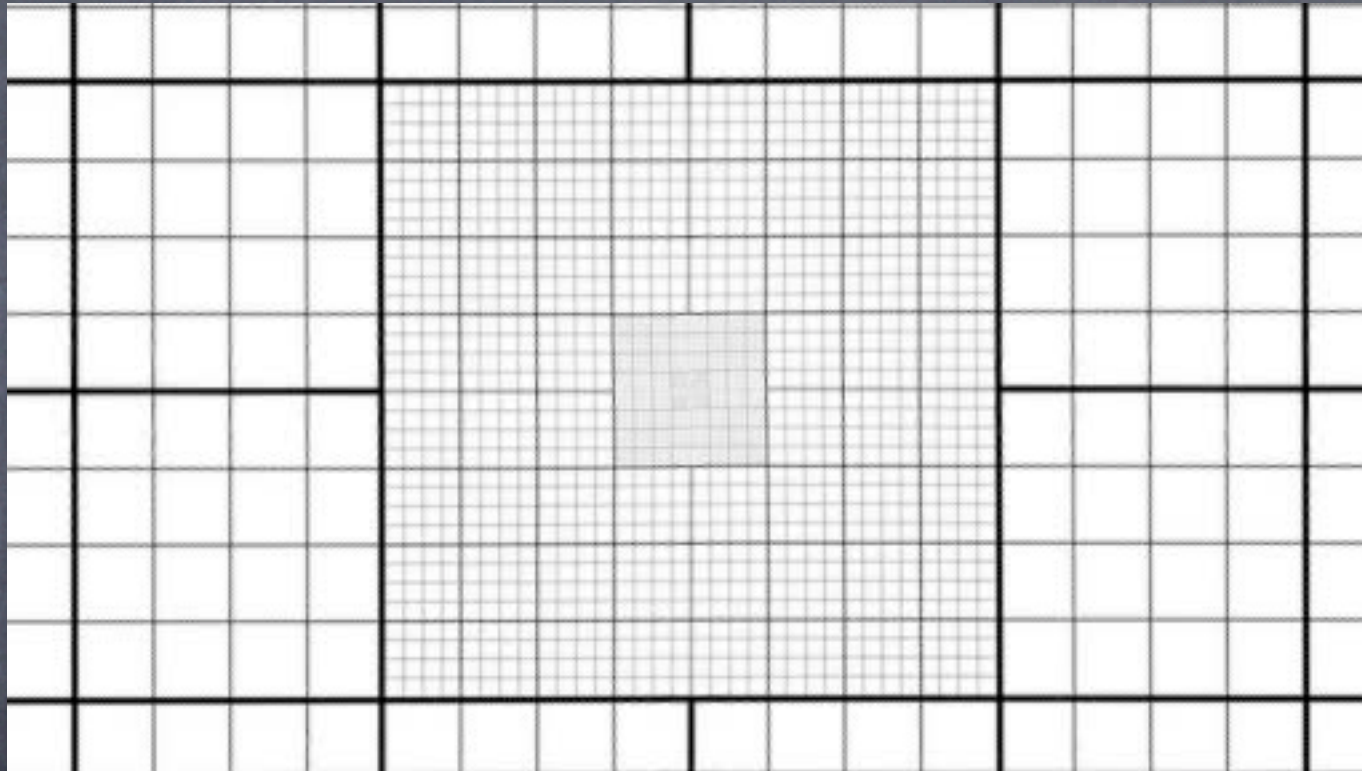
$w^\alpha$

←





# "L'UNIQUE BONNE" TORSION EN IMAGE

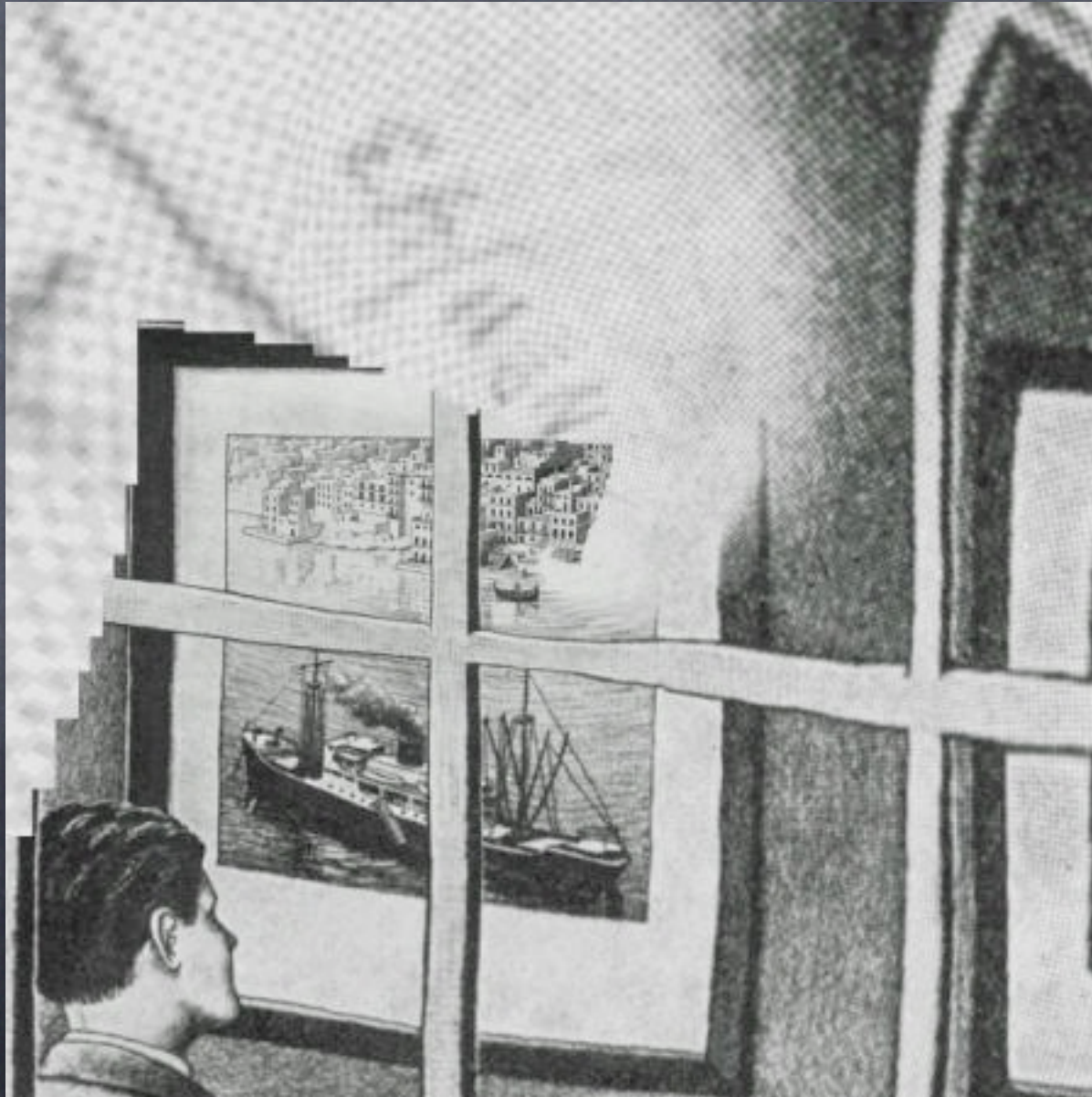




"ACHEVER" L'OEUVRE  
D'ESCHER



# REDRESSER LA LITHOGRAPHIE

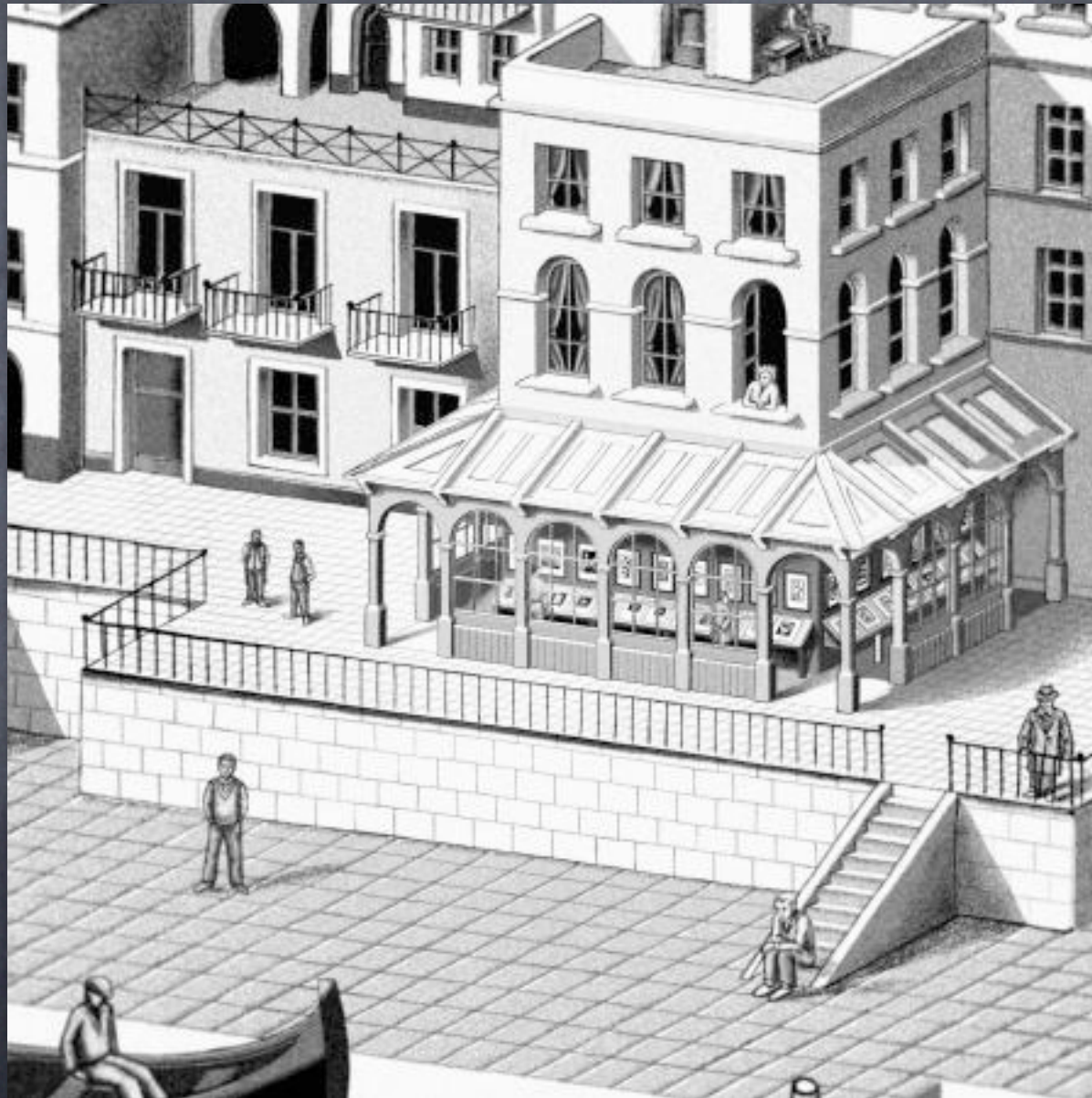


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# COMPRETER "L'ORIGINAL"

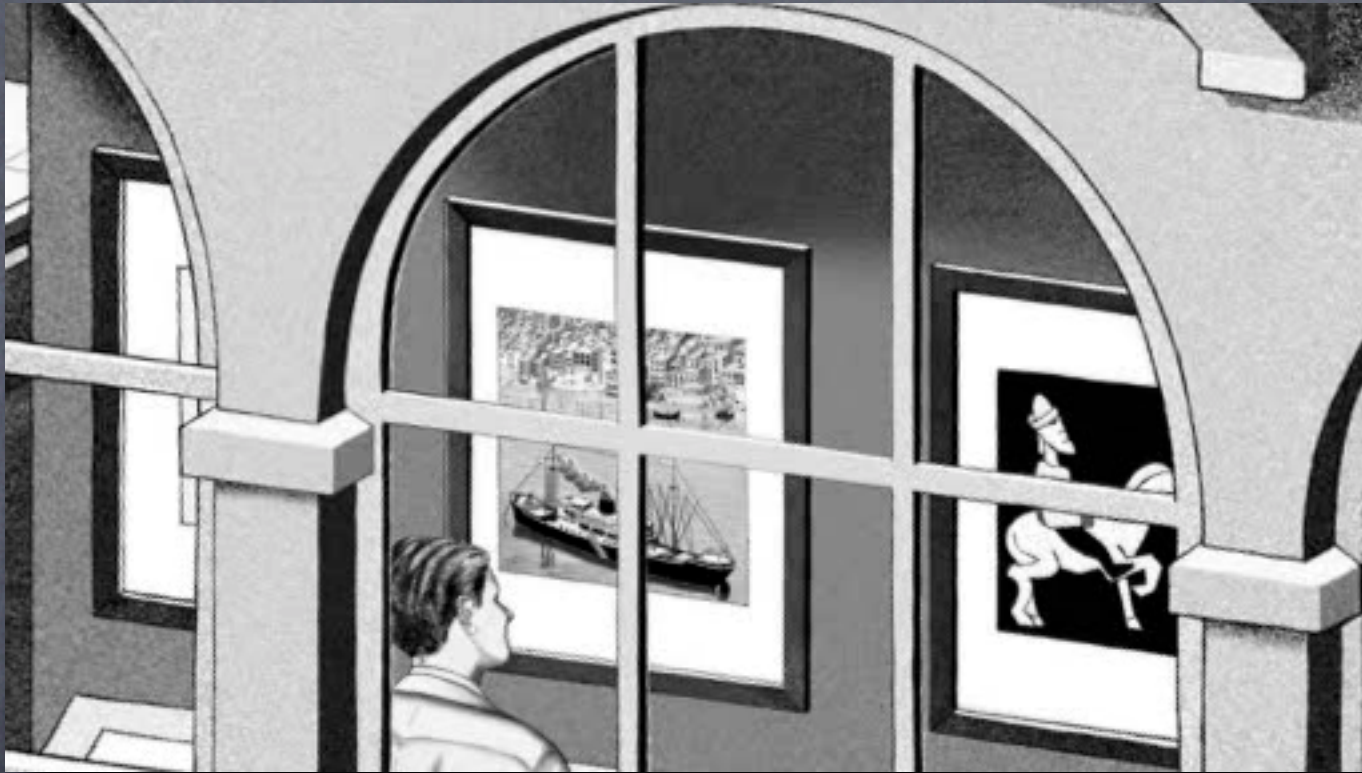


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# RESULTAT FINAL

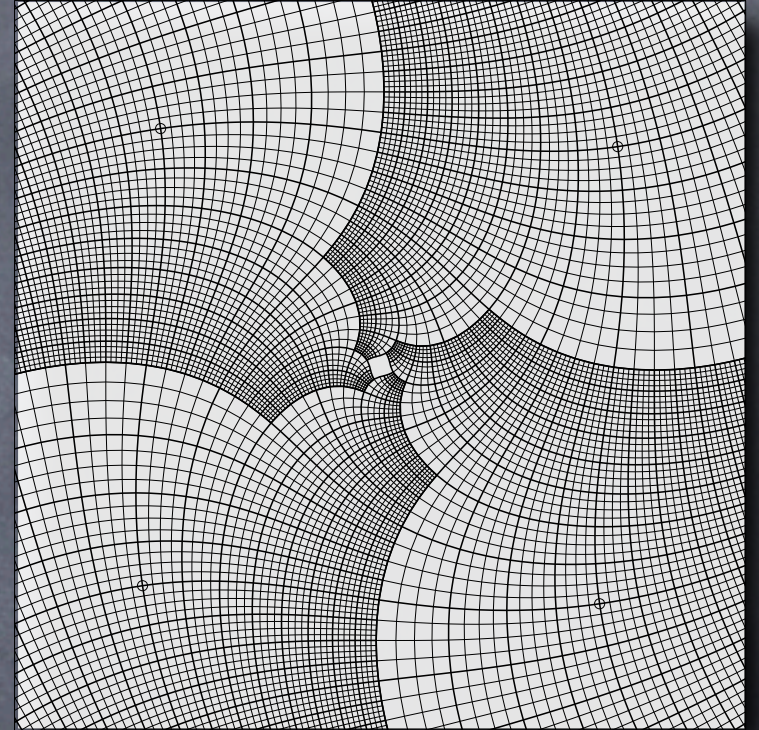
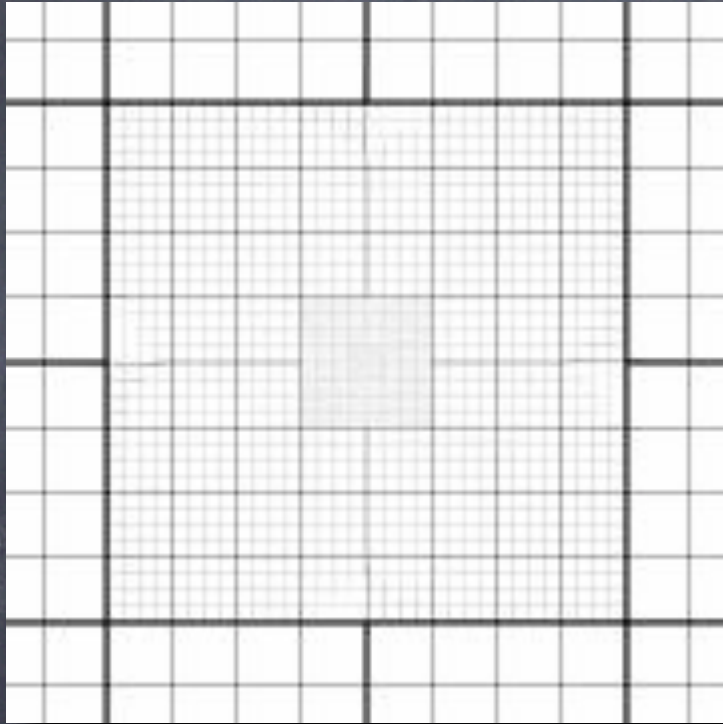


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# APPLIQUER LA TORSION





# APPLIQUER LA TORSION



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# zoom

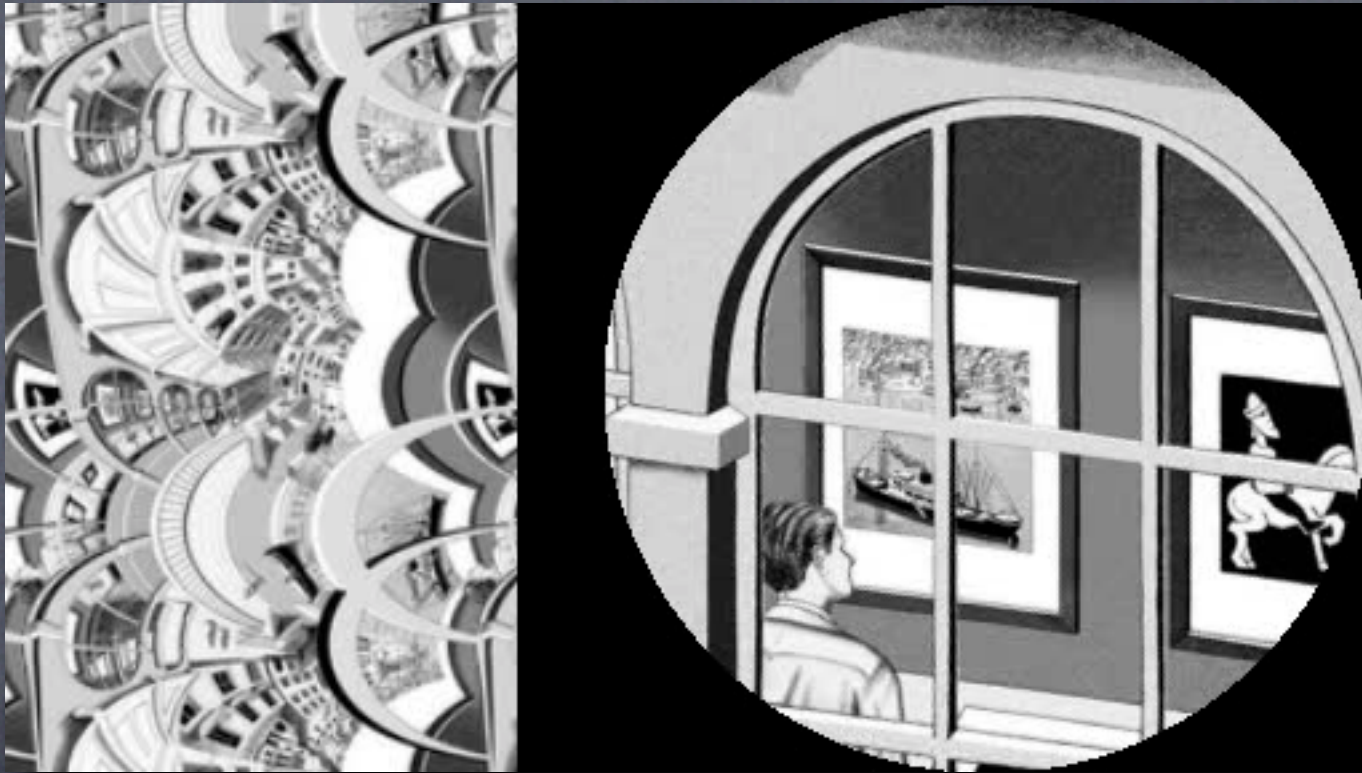


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# EN RÉSUMÉ



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Fin





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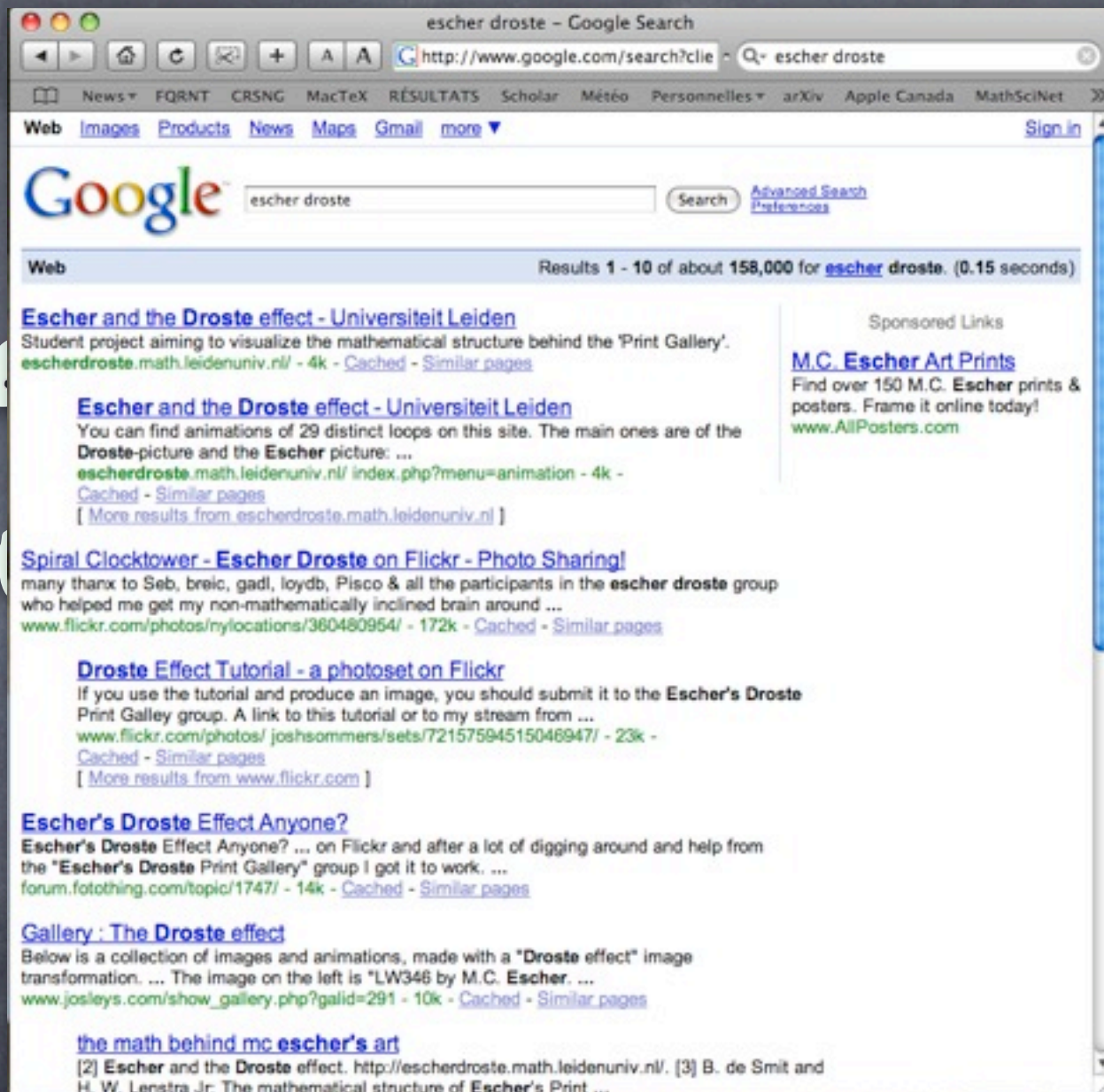


Voir:

- BEAG

- Google

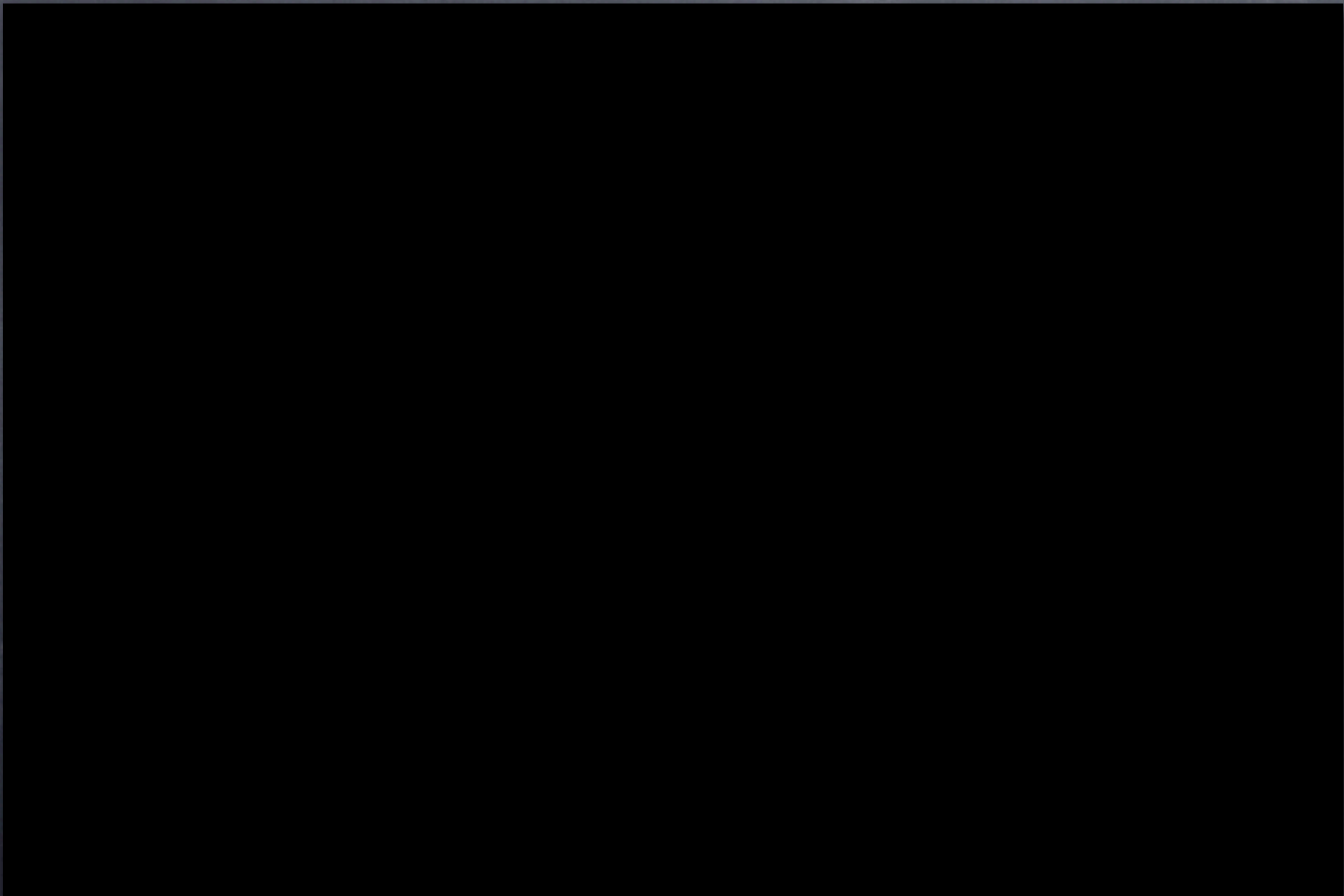
- ...



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