Les mathématiques au secours DE L'ART D'ESCHER


Relativité,
1953

Frwacois Bergerow, deft. Math, UGAM

Les mathématiques au secours DE L'ART D'ESCHER


Andrew Lipson et Daniel Shiu, 2000

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Les mathématiques au secours
DE L'ART D'ESCHER


Frewço is BERGEROW, DERT. MATH, UGAM
Tuesday, December 3, 13

Les mathématiques au secoves DE L'ART D'ESCHER


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Qui Est Escher

Maurits Cornelis Escher

$$
(1818-1972)
$$



Drawing Hands, 1948


Autoportrait, 1943

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Maison de ferme à Ravello, Rome 1931.
Frewcois BERGERON, DEPT. MATH, UGAM

cente,

Frwncois BeRgeraw, deft Math, UGAM


Chute,

Frwçois BERGEROW, DEPT. MATH, UGAM


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SOURCES D INSPIration

# $L^{\prime} A L H A M B R A$ (GRENADE, ESPAGNE.) 



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## UNE AMITIE DATANT DE 1954


H.S.M. Coxeter (1907-2001) Univ. Toronto


Plan hyperbolique

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# GÉOMÉTRIE HYPsßBOLIqUE 



Oystal symmetry and its generalizations
HSM Coxeter - Trans. Roya Soc. Canada (3), 1957, 1-

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## A LA ESCHER



Cercle limite III, 1959

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## Ruban de Moebius

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Ruban de Moebius II, 1963

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LART de la dÉformation


Balcon, 1945
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## Une Vielle TRAdition



Les ambassadeurs, Holbein (1533)

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Frwçois Bergerow, deft. Math, UGAM
"Prentententoonstelling" (M.C. EsCHER, 1956)


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## Hendrik w. Lenstra

Mathematical Institute
Universeit Leiden
Netherlands

Department of Mathematics
University of California at Berkeley
USA

Topics: Algebra, number theory, algorithms


Titles of major publications

- Factoring integers with elliptic curves Annals of Mathematics 126 (1987), 649-673.
- Algorithms in algebraic number meory, Bulletin of the American Mathematical Society 26 (1992), 211-244.
- The development of the number field sieve (with A.K. Lenstra), Lecture Notes in Mathematics 1554, Springer-Verlag, Heidelberg, 1993.
- Artin reciprocity and Mersenne primes (with P. Stevenhagen), Nieuw Archief voor Wiskunde (5) 1 (2000), 44-54.
- Flags and lattice basis reduction, European congress of mathematics, Birkhäuser, Basel, 2001.

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## Une Revue de

Fruncois Bergeraw, deft. Math, UGAM

# Mathematician Fills in a Blank for a Fresh Insight on Art 

By SARA ROBINSON

0n a flight to the Netherlands, Dr. Hendrik Lenstra, a mathematician, was leafing through an airline magazine when a picture of a lithograph by the Dutch artist M. C. Escher caught his eye.

Titled "Print Gallery," it provides a glimpse through a row of arching windows into an art gallery, where a man is gazing at a picture on the wall. The picture depicts a row of Mediterranean-style buildings with turrets and balconies, fronting a quay on the island of Malta.

As the viewer's eye follows the line of buildings to the right, it begins to bulge outward and twist downward, until it sweeps around to include the art gallery itself. In the center of the dizzying whorl of buildings, ships and sky, is a large, circular patch that Escher left blank. His signature is scrawled across it.

As Dr. Lenstra studied the print he found his attention returning again and again to that central patch, puzzling over the reason Escher had not filled it in. "I wondered whether if you continue the lines inward, if there's a mathematical problem that cannot be solved," he said. "More generally, I also wondered what the structure is behind the picture: how would I, as a mathematician, make a picture like that?"

# Artful Mathematics: The Heritage of M.C.Escher 

## Celebrating Mathematics Awareness Month

In recognition of the 2003 Mathematics Awareness Month theme "Mathematics and Art", this article brings together three different pieces about intersections between mathematics and the artwork of M. C. Escher. For more information about Mathematics Awareness Month, visit the website http:// mathforum.org/mam/03/. The site contains materials for organizing local celebrations of Mathematics Awareness Month.

## The Mathematical Structure of Escher's Print Gallery

B. de Smit and H. W. Lenstra Jr.

In 1956 the Dutch graphic artist Maurits Cornelis Escher (1898-1972) made an unusual lithograph with the title Prentententoonstelling. It shows a young man standing in an exhibition gallery, viewing a print of a Mediterranean seaport. As his eyes follow the quayside buildings shown on the print from left to right and then down, he discovers among them the very same gallery in which he is standing. A circular white patch in the middle of the lithograph contains Escher's monogram and signature.

What is the mathematics behind Prentententoonstelling? Is there a more satisfactory way of filling in the central white hole? We shall see that the lithograph can be viewed as drawn on a certain elliptic curve over the field of complex numbers and
B. de Smit and H. W. Lenstra Jr. are at the Mathematisch Instituut, Universiteit Leiden, the Netherlands. H. W. Lenstra also holds a position at the University of California, Berkeley. Their email addresses are desmit@math leidenuniv.nl and hwl@math. leidenuniv.n7.


Figure 1. Escher's lithograph "Prentententoonstelling" (1956).
deduce that an idealized version of the picture repeats itself in the middle. More precisely, it contains a copy of itself, rotated clockwise by $157.6255960832 \ldots$ degrees and scaled down by a factor of $22.5836845286 \ldots$

## Escher's Method

The best explanation of how Prentententoonstelling was made is found in The Magic Mirror of M. C. Escher by Bruno Ernst [1], from which the following quotations and all illustrations in this section are taken. Escher started "from the idea that it must...be possible to make an annular bulge," "a cyclic expansion...without beginning or end." The realization of this idea caused him "some almighty headaches." At first, he "tried to put his idea into

## LA DÉMARCHE D'ESCHER

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# Ce quion sait de la DÉMARCHE D'ESCHER 



## Une étude autosimilaire

Une grille de "torsion"
3) Son intention explicite de conserver l'apparence de "petits" carrés.


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## UNE ÉTUDE AUTO-SIMILAIRE



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## L'egeet d.esies



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## L'EFFET "VACHE qui RIt"



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## L'EFFET "UMMAGUMMA



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## Mathématisation

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## Nombeas ComPlexes



## Nombess Comprexes

$$
\begin{gathered}
z=x+i y \quad i=\sqrt{-1} \\
(x+i y)(u+i v)=(x u-y v)+i(x v+y u) \\
\exp (x+i y)=e^{x} \cos (y)+i e^{x} \sin (y) \\
\frac{d}{d x} \exp (x+i y)=e^{x} \cos (y)+i e^{x} \sin (y) \\
\frac{d}{d y} \exp (x+i y)=-e^{x} \sin (y)+i e^{x} \cos (y)
\end{gathered}
$$

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## Nombeas ComPlexes

$$
\begin{aligned}
w & =\cos (\theta)+i \sin (\theta) \\
& =\exp (i \theta)
\end{aligned}
$$



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## UNE IMAGE



$$
f(z)=f(256 z)
$$

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## LA GRille D'ESCHER

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## LA

"Torsion"


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# LA "TORSION" D'ASCHER (conserver lapparence des capeás) 



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## Une autre vieille teadition



Albrecht Dürer, "Undersweysung der Messung", 1527.

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## Une autre vieille teadition



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## Les Mathematiques DE LA TORSION

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## Transformations conformes

 (RESPGGTANT LES ANGLES) $h(x+i y)=u(x, y)+i v(x, y)$

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## Transformations conforms

 (RESPBGTANT LES ANGLES) $h(x+i y)=u(x, y)+i v(x, y)$

$$
\frac{d h}{d x}=-i \frac{d h}{d y}
$$

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## Transformations conformes

 (RESPGGTANT LES ANGLES)$$
\begin{aligned}
h(x+i y) & =u(x, y)+i v(x, y) \\
\frac{d h}{d x} & =-i \frac{d h}{d y}
\end{aligned}
$$



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## LOGARITHME

$\log (256)$



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## LOGARITHME



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## LOGARITHME / Ex PonentiElle <br> En Action



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THÉRÉME:
LES TRANSFORMations conformas Du tore vers le tore SONT DE LA FORME... Et seviensur de cette forme

Il n'y a qu'une et une seule transformation conforme qui satisfasse les contraintes imposées par Escher!

## "L'uniave bonne" Torsion



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"L'uniave bowne" Torsion En image


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## "ACHEVER" L'OGUVKE D'ESGHER

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Redresser la lithograhtie


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## COMRETER "L'eriginal"



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## RESULTAT FINAL



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## APPlieuer LA TORSIN



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## APPliquer LA TORSION



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## VERSION ESGHER



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## ZOOM



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## EN <br> Resumé



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Escher and the Droste effect - Universiteit Leiden
Student project aiming to visualize the mathematical structure behind the 'Print Gallery'. escherdroste.math leidenuniv.rl/ - 4k - Cached - Similar papes

Escher and the Droste effect-Universiteit Leiden
You can find animations of 29 distinct locps on this site. The main ones are of the Droste-picture and the Escher picture:
escherdroste math.leidenuniv.nil/ index.php?menu=animation - 4 k -
Cached-Similar peges
[More results from escherdrogte, math, loidonuriv.n] ]
Spiral Clocktower - Escher Droste on Flickr - Photo Sharingl
many tharx to Seb, breic, gadi, loydb, Pisco \& all the participants in the escher droste group who helpod me get my non-mathematically inclined brain around .
wow.flickr.com/photos/nylocations/380480954/ - 172k - Cached - Similar pages
Droste Effect Tutorial - a photoset on Flickr
If you use the tutorial and produce an image, you should submit it to the Escher's Droste
Print Galley group. A link to this tutorial or to my stream from
www.fickr.com/photos/ joshsommers/sets/72157594515046947/-23k
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[ More results from www.flickr com] ]
Escher's Droste Effect Anyone?
Escher's Droste Effect Anyone? ... on Flickr and after a lot of digging around and help from the "Escher's Droste Print Galery" group I got it to work. forum,fotothing.com/topic/1747/ - 14k - Cached - Simiar pages

Gallery: The Droste effect
Below is a collection of images and arimations, made with a "Droste effect" image transformation. ... The image on the left is "LW346 by M.C. Escher.
wow.josleys.com/show_gallery.php?galid=291 - 10k - Cachod - Similar poges
the math behind mc escher's art
[2] Escher and the Droste effect. httpol/escherdroste.math.leidenuniv.nl/, (3) B. de Smit and
H. W. Lenstra Jr. The mathematical structure of Escher's Print
M.C. Escher Art Prints Find over 150 M.C. Escher prints \& poaters. Frame it online today? www.AlPosters.com

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